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SHIPPING AND DISCOUNT ASPECTS IN OPTIMAL INVENTORY MANAGEMENT

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Abstract: There are many and complex models in inventory management. The novelty of the presented model is that it takes into consideration the actual aspects in shipping of needed objects or materials (number of trucks or other means of transportation, their capacity and availability, carrying durations and schedule) and the correlation between the costs and the ordered quantities (discounts offered by manufacturers and suppliers if ordered quantities exceed certain levels). The consumption of stored materials is supposed to be a random variable that follows a normal distribution. The optimal scheme was obtained by running a genetic algorithm implemented in *Cambrian v.1.0* software.

1. INTRODUCTION

Although, there are a lot of researches in the field of inventory management (we mentioned here only [2] [1] [4]), a comprehensive optimization model did not issue yet. The authors proposed themselves in [5] a model of the optimal management of the distribution networks. The difficulty of problem consists of the existence of a lot of factors that the manager should take into consideration for an optimal management. The novelty of the presented model is that it takes into consideration the actual aspects in shipping of needed objects or materials (number of trucks or other means of transportation, their capacity and availability, carrying durations and schedule) and the correlation between the costs and the ordered quantities (discounts offered by manufacturers and suppliers if ordered quantities exceed certain levels). The consumption of stored materials is supposed to be a random variable that follows a normal distribution. The optimal scheme was obtained by using a genetic algorithm (for further information on genetic algorithms, see [3]).

2. PROBLEM FORMULATION

Let assume that the fictitious *Giant Television Company* (GTC) is assembling CRT (cathode-ray tube) TV sets and the daily production is about 100 units. GTC buy the CRTs from *Delp Company* and ships them by their own transportation trucks. Our interest is focused of the CRT storage warehouse management. In table 1 the main input design data are presented (all data are fictive but realistic):

Table 1. Input data

Main data	Designation	Value
Mean (daily) CRT consumption	μ_{CRT}	100 units
Standard deviation (daily) of CRT consumption	σ_{CRT}	10 units
Number of days of the cycle	N	100 days
Carrying capacity of trucks of type 1	tc_1	55 units
Carrying capacity of trucks of type 2	tc_2	70 units
Number of trucks of type 1	Nmt_1	6 trucks
Number of trucks of type 2	Nmt_2	4 trucks
Initial CRT stock	$s(0)$	213 units
Shipping duration (GTC – Delp – GTC)	td	2 days

In order to obtain an optimal scheme of buying, shipping and storage of the CRT we assume the consumption scenario presented in table 2 (z = day number, and $c(z)$ = number of CRTs consumed by GTC in the day z). The values in table 2 are randomly generated and follow a normal law (mean: $\mu = 100$ units; standard deviation: $\sigma = 10$ units).

Table 2. Consumption scenario

z	c(z)	z	c(z)	Z	c(z)	z	c(z)	z	c(z)
1	70	21	123	41	112	61	97	81	103
2	102	22	99	42	106	62	87	82	97
3	91	23	88	43	117	63	105	83	106
4	109	24	74	44	114	64	84	84	103
5	102	25	76	45	101	65	76	85	102
6	99	26	97	46	89	66	114	86	96
7	96	27	101	47	99	67	94	87	90
8	113	28	102	48	93	68	94	88	92
9	109	29	103	49	111	69	102	89	98
10	107	30	103	50	92	70	105	90	109
11	91	31	90	51	108	71	110	91	100
12	111	32	104	52	110	72	106	92	123
13	106	33	99	53	127	73	100	93	107
14	100	34	96	54	134	74	92	94	96
15	95	35	84	55	103	75	107	95	90
16	78	36	103	56	97	76	99	96	104
17	87	37	108	57	94	77	99	97	100
18	97	38	109	58	95	78	116	98	85
19	90	39	100	59	110	79	107	99	105
20	90	40	95	60	80	80	88	100	100

As one could see in table 3 *Delp Company* practices a very interesting discount policy according to the ordered CRT quantities. Note that the price per unit without discount of the CRT is *PPU* = 500 RON.

Table 3. *Delp* discount policy

Ordered quantity	Discount	Price per unit (RON)
Up to 100 units	0%	PPU = 500
101 … 200 units	3%	485
201 … 300 units	6%	470
301 … 400 units	9%	455
Over to 401 units	12%	440

Obviously, GTC has to buy, then has to ship and store the CRTs and eventually, has to find an optimal balance between the contradictory aspects of acquisition, shipping and storage costs. Buying a huge quantity from the beginning seems to be not a very good idea since the storage costs will be increased very much. Also, buying equal quantities at equal periods of time means to not exploit the advantages of the discounts offered by the seller company.

Regarding to the shipping costs is interesting to know in which days and how many trucks of which type the manager has to send after CRTs. In table 4 the shipping and storage costs per unit are presented.

Table 4. Shipping and storage costs per unit

Costs	Designation	Value
Shipping cost for trucks of type 1	ct_1	8.1 RON/unit
Shipping cost for trucks of type 2	ct_2	5.8 RON/unit
Daily storage cost	sc	0.3 RON/unit/day

3. OPTIMAL DESIGN OF THE ACQUISITION-SHIPPING-STORAGE SCHEME

In this model three categories of costs were taking into account: acquisition costs, shipping costs and storage costs. All these three costs reach different amount of cash according to the acquisition-shipping-storage scheme and in order to reach the optimum we propose the following optimization program:

3.1. GENES (VARIABLES)

After a closely analysis of the optimization problem we proposed N = 100 genes (as the number of the days of the working cycle), that is 100 codes of integer values in the range of 0 … $Nmt_1 \cdot Nmt_2 + Nmt_1 + Nmt_2 = 34$. Each of these 100 numbers $Gena(z)$ is carrying in fact two values: $nmt_1(z)$ = the number of trucks of type 1 used in the day z, and $nmt_2(z)$ = the number of trucks of type 2 used in the same day z.

$$Gena(z) = (Nmt_2 + 1) \cdot nmt_1(z) + nmt_2(z), \quad z = \overline{1, N} \tag{1}$$

The de-codification process consists of dividing $Gena(z)$ to (Nmt_2+1) and assign the quotient to $nmt_1(z)$ and the reminder to $nmt_2(z)$. In this manner the number of necessary genes is reduced to a half. If the values of genes are known it becomes very easy to compute the daily acquisitions, i. e. $a(z)$ and daily stocks $s(z)$:

$$a(z) = tc_1 \cdot nmt_1(z) + tc_2 \cdot nmt_2(z), \quad z = \overline{1, N} \tag{2}$$

$$s(z) = s(z-1) + a(z) - c(z), \quad z = \overline{1, N} \tag{3}$$

3.2. OJECTIVE FUNCTION

The objective function chosen for this optimization model is the total cost per working cycle and it consist of acquisition costs, shipping costs and storage costs.

$$TotalCost = AcquisitionCost + ShippingCost + StorageCost \tag{4}$$

where:

$$AcquisitionCost = \sum_{z=1}^{N} AcC(z) = \sum_{z=1}^{N} PPU \cdot a(z) \cdot (1 - discount(z)) \tag{5}$$

$$ShippingCost = \sum_{z=1}^{N} ShC(z) = \sum_{z=1}^{N} (tc_1 \cdot ct_1 \cdot nmt_1(z) + tc_2 \cdot ct_2 \cdot nmt_2(z)) \tag{6}$$

$$StorageCost = \sum_{z=1}^{N} StC(z) = \sum_{z=1}^{N} sc \cdot s(z) \tag{7}$$

In equation (5) $discount(z)$ is the daily discount offered by $Delp$ according to the daily acquisition quantity $a(z)$.

3.3. CONSTRAINTS

The solution of the optimization program has to satisfy a large number of constraints (300, in fact), as follows:

C001 – C100: Daily stocks has to exceed a so called *SafetyStock* = 200 units.

$$s(z) \geq SafetyStock, \quad z = \overline{1, N} \tag{8}$$

C101 – C298: Since the shipping duration td is longer than a single day it is mandatory that in any td consecutively days the sum of the number of the trucks (of any type) has to be less or equal to the number of existing trucks.

$$\sum_{k=z}^{z+td-1} nmt_1(k) \leq Nmt_1, \quad z = \overline{1, N-td+1} \tag{9}$$

$$\sum_{k=z}^{z+td-1} nmt_2(k) \leq Nmt_2, \quad z = \overline{1, N-td+1} \tag{10}$$

C299: The total acquisition has to be sufficient to the needs of the working cycle:

$$s(0) + \sum_{z=1}^{N} a(z) - \sum_{z=1}^{N} c(z) \geq 0 \tag{11}$$

C300: The total acquisition has to not exceed the needs of the working cycle:

$$s(0) + \sum_{z=1}^{N} a(z) - \sum_{z=1}^{N} c(z) \leq ct_1 - 1 \tag{12}$$

3.4. RESULTS

The optimization program was solved using *Cambrian v.1.0* software belonging to the Optimal Design Centre of the Technical University of Cluj-Napoca. In tables 5-7 are presented the main results, obtained after 20,000 generations.

Table 5. Daily acquisition, consumption and stock

z	nmt_1	nmt_2	a(z)	c(z)	s(z)	z	nmt_1	nmt_2	a(z)	c(z)	s(z)
1	0	1	70	70	213	51	0	0	0	108	1,084
2	3	1	235	102	346	52	1	0	55	110	1,029
3	1	0	55	91	310	53	2	4	390	127	1,292
4	0	0	0	109	201	54	0	0	0	134	1,158
5	2	0	110	102	209	55	0	0	0	103	1,055
6	4	4	500	99	610	56	2	0	110	97	1,068
7	0	0	0	96	514	57	1	0	55	94	1,029
8	0	0	0	113	401	58	2	0	110	95	1,044
9	4	4	500	109	792	59	1	0	55	110	989
10	1	0	55	107	740	60	1	0	55	80	964
11	0	0	0	91	649	61	1	0	55	97	922
12	6	1	400	111	938	62	0	0	0	87	835
13	0	0	0	106	832	63	0	0	0	105	730
14	0	0	0	100	732	64	0	0	0	84	646
15	1	0	55	95	692	65	1	0	55	76	625
16	1	0	55	78	669	66	0	0	0	114	511
17	0	0	0	87	582	67	0	0	0	94	417
18	0	0	0	97	485	68	1	0	55	94	378

z	nmt_1	nmt_2	a(z)	c(z)	s(z)	z	nmt_1	nmt_2	a(z)	c(z)	s(z)
19	2	0	110	90	505	69	0	0	0	102	276
20	2	0	110	90	525	70	4	4	500	105	671
21	0	0	0	123	402	71	2	0	110	110	671
22	2	0	110	99	413	72	1	0	55	106	620
23	0	0	0	88	325	73	0	0	0	100	520
24	0	0	0	74	251	74	0	0	0	92	428
25	4	4	500	76	675	75	1	0	55	107	376
26	0	0	0	97	578	76	0	0	0	99	277
27	0	0	0	101	477	77	3	1	235	99	413
28	2	4	390	102	765	78	0	0	0	116	297
29	1	0	55	103	717	79	2	0	110	107	300
30	2	0	110	103	724	80	1	0	55	88	267
31	0	0	0	90	634	81	5	4	555	103	719
32	1	0	55	104	585	82	1	0	55	97	677
33	0	0	0	99	486	83	0	0	0	106	571
34	0	0	0	96	390	84	0	0	0	103	468
35	0	0	0	84	306	85	0	0	0	102	366
36	0	0	0	103	203	86	0	0	0	96	270
37	3	2	305	108	400	87	3	2	305	90	485
38	0	0	0	109	291	88	0	0	0	92	393
39	1	0	55	100	246	89	6	4	610	98	905
40	4	3	430	95	581	90	0	0	0	109	796
41	0	0	0	112	469	91	0	0	0	100	696
42	4	0	220	106	583	92	2	4	390	123	963
43	2	4	390	117	856	93	0	0	0	107	856
44	4	0	220	114	962	94	0	0	0	96	760
45	1	0	55	101	916	95	0	0	0	90	670
46	1	0	55	89	882	96	0	0	0	104	566
47	4	3	430	99	1,213	97	1	0	55	100	521
48	2	0	110	93	1,230	98	0	0	0	85	436
49	1	0	55	111	1,174	99	0	0	0	105	331
50	2	0	110	92	1,192	100	0	0	0	100	231

Table 6. Daily acquisition, shipping and storage costs

z	AcC(z)	ShC(z)	StC(z)	z	AcC(z)	ShC(z)	StC(z)
1	35,000	406	63.9	51	0	0	325.2
2	110,450	1742.5	103.8	52	27,500	445.5	308.7
3	27,500	445.5	93	53	177,450	2,515	387.6
4	0	0	60.3	54	0	0	347.4
5	53,350	891	62.7	55	0	0	316.5
6	220,000	3,406	183	56	53,350	891	320.4
7	0	0	154.2	57	27,500	445.5	308.7
8	0	0	120.3	58	53,350	891	313.2
9	220,000	3,406	237.6	59	27,500	445.5	296.7

z	AcC(z)	ShC(z)	StC(z)	z	AcC(z)	ShC(z)	StC(z)
10	27,500	445.5	222	60	27,500	445.5	289.2
11	0	0	194.7	61	27,500	445.5	276.6
12	182,000	3,079	281.4	62	0	0	250.5
13	0	0	249.6	63	0	0	219
14	0	0	219.6	64	0	0	193.8
15	27,500	445.5	207.6	65	27,500	445.5	187.5
16	27,500	445.5	200.7	66	0	0	153.3
17	0	0	174.6	67	0	0	125.1
18	0	0	145.5	68	27,500	445.5	113.4
19	53,350	891	151.5	69	0	0	82.8
20	53,350	891	157.5	70	220,000	3,406	201.3
21	0	0	120.6	71	53,350	891	201.3
22	53,350	891	123.9	72	27,500	445.5	186
23	0	0	97.5	73	0	0	156
24	0	0	75.3	74	0	0	128.4
25	220,000	3,406	202.5	75	27,500	445.5	112.8
26	0	0	173.4	76	0	0	83.1
27	0	0	143.1	77	110,450	1,742.5	123.9
28	177,450	2515	229.5	78	0	0	89.1
29	27,500	445.5	215.1	79	53,350	891	90
30	53,350	891	217.2	80	27,500	445.5	80.1
31	0	0	190.2	81	244,200	3,851.5	215.7
32	27,500	445.5	175.5	82	27,500	445.5	203.1
33	0	0	145.8	83	0	0	171.3
34	0	0	117	84	0	0	140.4
35	0	0	91.8	85	0	0	109.8
36	0	0	60.9	86	0	0	81
37	138,775	2,148.5	120	87	138,775	2,148.5	145.5
38	0	0	87.3	88	0	0	117.9
39	27,500	445.5	73.8	89	268,400	4297	271.5
40	189,200	3,000	174.3	90	0	0	238.8
41	0	0	140.7	91	0	0	208.8
42	103,400	1,782	174.9	92	177,450	2,515	288.9
43	177,450	2,515	256.8	93	0	0	256.8
44	103,400	1,782	288.6	94	0	0	228
45	27,500	445.5	274.8	95	0	0	201
46	27,500	445.5	264.6	96	0	0	169.8
47	189,200	3,000	363.9	97	27,500	445.5	156.3
48	53,350	891	369	98	0	0	130.8
49	27,500	445.5	352.2	99	0	0	99.3
50	53,350	891	357.6	100	0	0	69.3

In figure 1 the levels of the daily consumption, acquisitions, and stock are presented. One could observe that there are six acquisition campaigns. The largest one is somehow situated in the middle of the cycle, with a peak of 1,300 units in stock. Obviously, these concentrated campaigns are a result of the discount policy of the

supplier. The largest campaign is not situated at the earlier stages of the acquisition cycle because in such case the storage costs will increase very much.

Table 7. Acquisition, shipping, storage, and total costs per cycle

Type cost	Costs
Acquisition cost	4,594,900
Shipping cost	72,265.50
Storage cost	18,732.90
Total cost	**4,685,898.40**

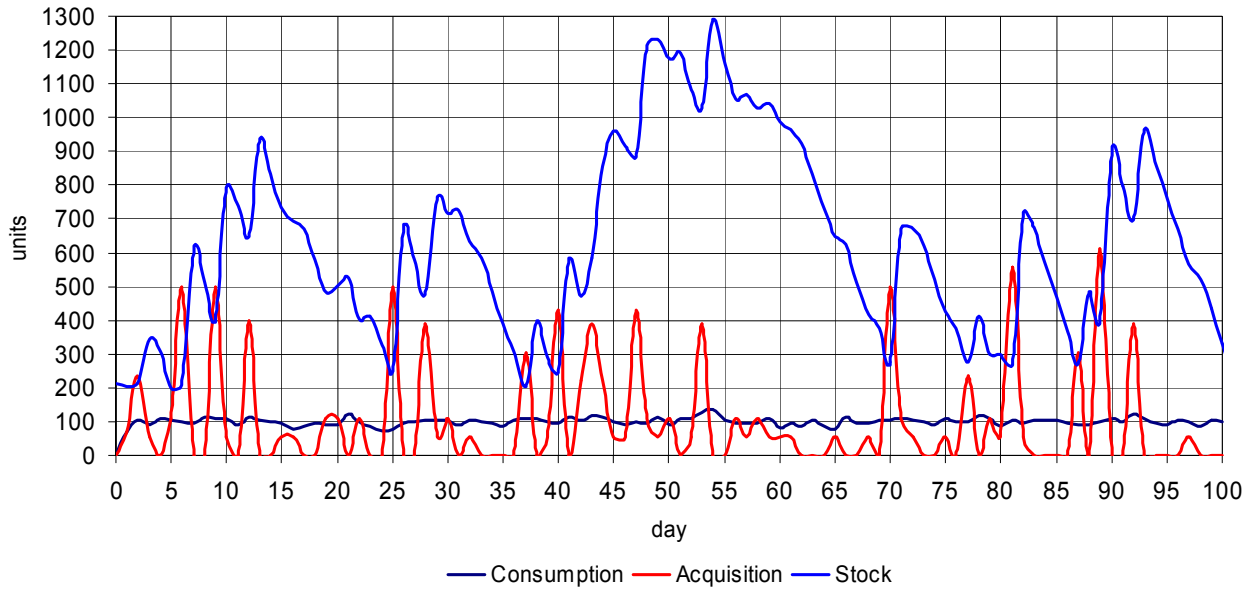

Figure 1. Daily consumption, acquisition, and stock

In figure 2 the aggregate consumption and the aggregate acquisition levels are presented.

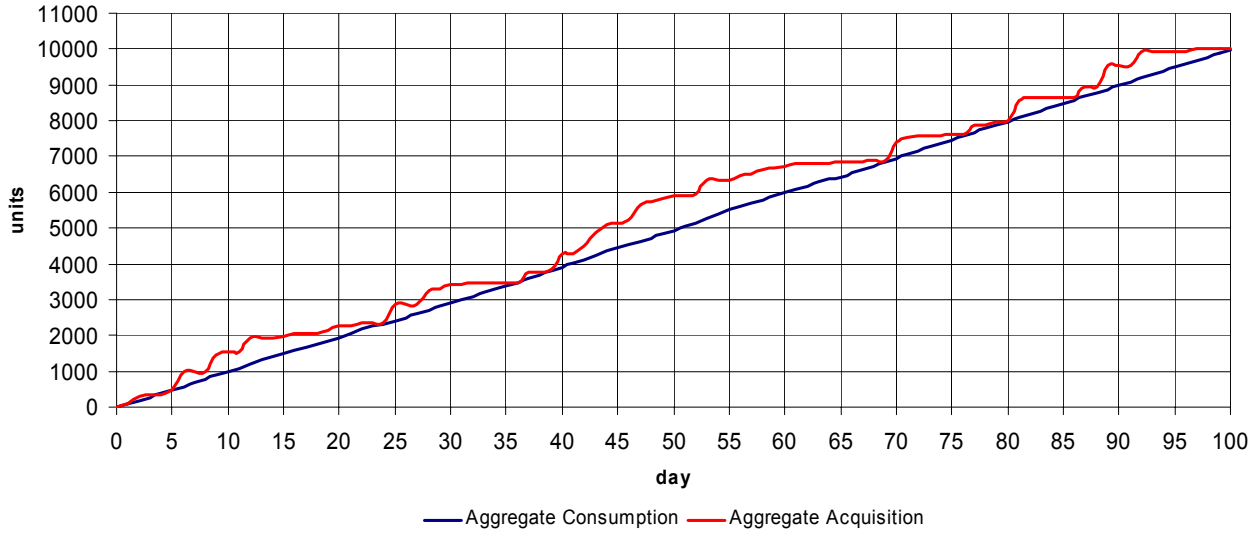

Figure 2. Aggregate consumption and acquisition

4. CONCLUSIONS

The total number of the acquired CRTs is 9977 units. If this quantity would be acquired in small lots of about 100 units daily (as Just in Time system suggests, since the mean consumption is in the same range) the total amount spent by GTC for the CRTs would be 500 × 9977 = 4,988,500 RON (because *Delp* do not offer discounts for small bought quantities). The advantage of this strategy would be that there is no storage cost involved in this case. The disadvantage consists of the fact that obviously the trucks will be not used at their full capacity. At this amount we have to add the transportation costs and we obtain 5,060,765.50 RON.

The whole amount of money which GTC has to spend according to the above obtained optimal scheme is 4,685,898.40 RON. It is important to mention here that this amount includes all the considered types of costs (i.e. acquisition costs, shipping costs and storage costs) as one could see in table 7.

The difference between these amounts of money is 374,867.10 RON that means a significant savings of about 7.4 %. Undoubtedly, this economy is very large and attractive for the GTC' managers.

Further researches have to focus on other scheme of shipping, with another means of transportation and on other systems of goods acquisition and payment (as the payment in advance for the not yet manufactured goods in order to benefit of a very large discount).

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