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A BIOMECANICAL MODEL OF THE BACKWARD LONGSWING ON FIXED BAR IN GYMNASTICS

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Abstract

In this paper we present a mathematical model for a gymnast to the backward longswing on fixed bar. Acted by forces and moments both external and internal, the sportsman tries to perform some elements of gymnastics. There are three kinds of generalized force: the gravitational forces, the forces induced by the man to realize the exercise and the biologic forces performed by the human body to avoid some non-usual positions.

1. INTRODUCTION

We consider a gymnast symbolized by *3* bars: *OA, AB* and *BC* (the significations are: hands, body and feet) (see Fig. 1).

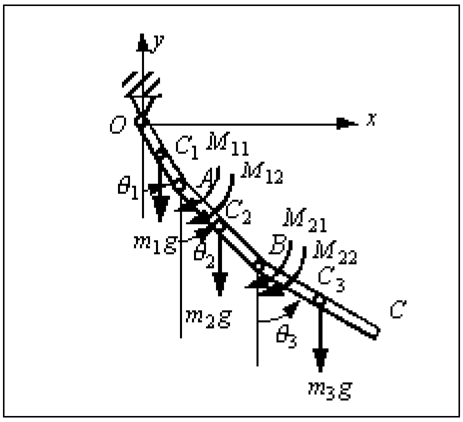

Fig. 1. The mechanical model for a gymnast

The bar *AB* has the mass m_1, the weight center C_1 at the distance $OA = r_1$, the length $OA = l_1$ and the inertial moment J_1 with respect to C_1. Analogously, for the bars *AB* and *BC* the following parameters are known: m_2, m_3, J_2, J_3, l_2, l_3, r_2 and r_3.

Moments M_{11} and M_{21} are due to the gymnast's wish to perform some elements. They write:

$$M_{i1}(t) = \begin{cases} m_{i1}^{1} & \text{for } t \in I_{i1}^{1} \\ \dots\dots\dots\dots & \\ m_{i1}^{j} & \text{for } t \in I_{i1}^{j} \\ \dots\dots\dots\dots & \\ m_{i1}^{k1} & \text{for } t \in I_{i1}^{k1} \\ 0 & \text{otherwise} \end{cases} \qquad (1)$$

where I_{i1}^{k1} are real disjoint intervals,

$$I_{i1}^{j} \cap I_{i1}^{l} = \Phi, \quad \text{for } j \neq l \qquad (2)$$

the number of these intervals being k_1. In the previous relations, t stands for time, and m_{i1} are constant moments, $i = 1, 2$.

In figure 2 we have drawn an example for the moments $M_{i1}(t)$.

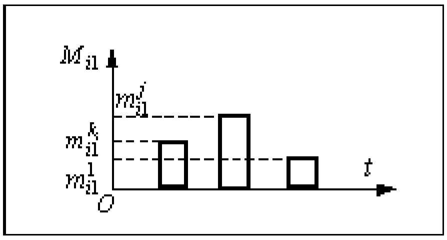

Fig. 2. An example of usual variation $M_{i1} = M_{i1}(t)$

The moments M_{i2} have as cause the human body's reaction to protect itself against non-normal position. We consider that these moments can read
with C_{i1}, C_{i2} constants, $i = 1, 2$, and the functions f_{1i} and f_{2i} write

$$f_{1i} = \begin{cases} 1 & \alpha_i \le \theta_{i+1} - \theta_i \le \alpha_i^{'} \\ 0 & otherwise \end{cases} \tag{4}$$

$$f_{2i} = \begin{cases} 1 & \beta_i \le \dot{\theta}_{i+1} - \dot{\theta}_i \le \beta_i^{'} \\ 0 & otherwise \end{cases} \tag{5}$$

In the previous relations $\alpha_i, \alpha'_i, \beta_i, \beta'_i$, $i = 1,2$ are physiological constants.

A simplified variant of these conditions is

$$\begin{aligned} &- \text{if } \theta_{i+1} \ge \theta_i \text{ and } \dot{\theta}_{i+1} \ge \dot{\theta}_i \text{ then } \theta_{i+1} = \theta_i \text{ and } \dot{\theta}_{i+1} = \dot{\theta}_i \\ &- \text{if } \theta_{i+1} \ge \theta_i \text{ then } \theta_{i+1} = \theta_i \\ &- \text{if } \theta_{i+1} \le \theta_i - \pi \text{ and } \dot{\theta}_{i+1} \le \dot{\theta}_i \text{ then } \theta_{i+1} = \theta_i - \pi \text{ and } \dot{\theta}_{i+1} = \dot{\theta}_l \\ &- \text{if } \theta_{i+1} \le \theta_i - \pi \text{ then } \theta_{i+1} = \theta_i \text{ and } \dot{\theta}_{i+1} = \dot{\theta}_l - \pi \end{aligned} \tag{6}$$

2. MOVING EQUATIONS

These equations are deduced with the aid of Lagrange's equation. From Euler-Lagrange formalism, we have

$$\frac{d}{dt}\left(\frac{\partial E_c}{\partial \dot{\theta}_i}\right) - \frac{\partial E_c}{\partial \theta_i} = Q_i \tag{7}$$

where Q_i are the generalized forces given by the following expressions:

$$\begin{aligned} Q_1 &= -(m_1 r_1 + m_2 l_1 + m_3 l_1) g \sin\theta_1, \\ Q_2 &= -(m_2 r_2 + m_3 l_2) g \sin\theta_2 - M_{11} - M_{12}, \\ Q_3 &= -(m_3 r_3) g \sin\theta_3 - M_{21} - M_{22} \end{aligned} \tag{8}$$

and E_c is the kinetic energy which writes

$$E_c = \frac{1}{2}\left(J_1\dot{\theta}_1^2 + m_1 v_{C_1}^2 + J_2\dot{\theta}_2^2 + m_2 v_{C_2}^2 + J_3\dot{\theta}_3^2 + m_3 v_{C_3}^2\right) \tag{9}$$

where

$$\begin{aligned} v_{C_1}^2 &= r_1^2\dot{\theta}_1^2, \\ v_{C_2}^2 &= l_1^2\dot{\theta}_1^2 + r_2^2\dot{\theta}_2^2 + 2l_1 r_2\dot{\theta}_1\dot{\theta}_2\cos(\theta_2 - \theta_1), \\ v_{C_2}^2 &= l_1^2\dot{\theta}_1^2 + l_2^2\dot{\theta}_2^2 + r_3^2\dot{\theta}_3^2 + 2l_1 l_2\dot{\theta}_1\dot{\theta}_2\cos(\theta_2 - \theta_1) + 2l_1 r_3\dot{\theta}_1\dot{\theta}_3\cos(\theta_3 - \theta_1) + 2l_2 r_3\dot{\theta}_2\dot{\theta}_3\cos(\theta_3 - \theta_2) \end{aligned} \tag{10}$$

Noting

$$\begin{aligned}
&a_{11} = J_1 + m_1 r_1^2 + (m_2 + m_3) l_1^2 \\
&a_{12} = a_{21} = (m_2 r_2 + m_3 l_2) l_1 \cos(\theta_2 - \theta_1) \\
&a_{13} = a_{31} = m_3 r_3 l_1 \cos(\theta_3 - \theta_1) \\
&a_{22} = J_2 + m_2 r_2^2 + m_3 l_2^2 \\
&a_{23} = a_{32} = m_3 r_3 l_2 \cos(\theta_3 - \theta_2) \\
&a_{33} = J_3 + m_3 r_3^2 \\
&b_{11} = b_{22} = b_{33} = 0 \\
&b_{12} = -b_{21} = -(m_2 r_2 + m_3 l_2) l_1 \sin(\theta_2 - \theta_1) \\
&b_{13} = -b_{31} = m_3 r_3 l_1 \sin(\theta_3 - \theta_1) \\
&b_{23} = -b_{32} = m_3 r_3 l_2 \sin(\theta_3 - \theta_2) \\
&k_1 = -(m_1 r_1 + m_2 l_1 + m_3 l_1) g \sin\theta_1 \\
&k_2 = -(m_2 r_2 + m_3 l_2) g \sin\theta_2 - M_{11} - M_{12} \\
&k_3 = -m_3 r_3 g \sin\theta_3 - M_{21} - M_{22}
\end{aligned} \tag{11}$$

$$[A] = \lfloor a_{ij} \rfloor_{i,j=\overline{1,3}} \qquad [B] = \lfloor b_{ij} \rfloor_{i,j=\overline{1,3}} \qquad [K] = [k_i]_{i=\overline{1,3}} \tag{12}$$

$$[\dot{\theta}] = [\dot{\theta}_i]_{i=\overline{1,3}} \qquad [\ddot{\theta}] = [\ddot{\theta}_i]_{i=\overline{1,3}} \tag{13}$$

the moving equations write

$$[A] \cdot [\ddot{\theta}] + [B] \cdot [\theta \cdot] = [K] \tag{14}$$

This relation can be seen as a linear system of three equations with three unknowns: $\ddot{\theta}_1, \ddot{\theta}_2, \ddot{\theta}_3$. The solution is

$$[\ddot{\theta}] = [A]^{-1}[B] \cdot [\theta \cdot] + [A]^{-1}[K] \tag{15}$$

3. NUMERICAL SIMULATION

Relation (15) can be written as

$$\begin{cases}
\ddot{\theta}_1 = H_1(\theta_1, \theta_2, \theta_3, \dot{\theta}_1, \dot{\theta}_2, \dot{\theta}_3) \\
\ddot{\theta}_2 = H_2(\theta_1, \theta_2, \theta_3, \dot{\theta}_1, \dot{\theta}_2, \dot{\theta}_3) \\
\ddot{\theta}_3 = H_3(\theta_1, \theta_2, \theta_3, \dot{\theta}_1, \dot{\theta}_2, \dot{\theta}_3)
\end{cases} \tag{16}$$

where the notations are evident.

Noting:

$$\begin{aligned}
&\xi_1 = \theta_1 \qquad \xi_2 = \theta_2 \qquad \xi_3 = \theta_3 \\
&\xi_4 = \dot{\theta}_1 \qquad \xi_5 = \dot{\theta}_2 \qquad \xi_6 = \dot{\theta}_7
\end{aligned} \tag{17}$$

one obtain from (16)

$$\begin{cases}
\dfrac{d\xi_1}{dt} = \xi_4 \qquad \dfrac{d\xi_4}{dt} = H_1(\xi_1, \xi_2, \xi_3, \xi_4, \xi_5, \xi_6) \\
\dfrac{d\xi_2}{dt} = \xi_5 \qquad \dfrac{d\xi_5}{dt} = H_2(\xi_1, \xi_2, \xi_3, \xi_4, \xi_5, \xi_6) \\
\dfrac{d\xi_3}{dt} = \xi_6 \qquad \dfrac{d\xi_6}{dt} = H_3(\xi_1, \xi_2, \xi_3, \xi_4, \xi_5, \xi_6)
\end{cases} \tag{18}$$

This system can be integrated by Runge-Kutta fourth order method.

Simulation was performed for the following parameters:

- m_1 = 8[kg], m_2 = 50[kg], m_3 = 16[kg];
- l_1 = 0.5[m], l_2 = 0.5[m], l_3 = 0.7[m];
- r_1 = 0.25[m], r_2 = 0.25[m], r_3 = 0.35[m];
- J_1 = 0.1666[kgm2], J_2 = 1.0416[kgm2], J_3 = 0.6533[kgm2].

The initial conditions are:

$$\theta_1^o = \xi_1^o = 3.124139[rad], \quad \theta_2^o = \xi_2^o = 3.124139[rad], \quad \theta_1^o = \xi_1^o = 3.124139[rad]$$

$$\dot{\theta}_1^o = \xi_4^o = 0[rad], \quad \dot{\theta}_2^o = \xi_5^o = 0[rad], \quad \dot{\theta}_1^o = \xi_6^o = 0[rad],$$

The constraints are considered to be offered by the relation (6). In these conditions the constraints impose only the avoiding of non-natural positions of the human body. In fact, the human body cannot reach these limit positions instantaneous and it cannot correct it in such situations. Biologically speaking, there exist zones in the space of all possible positions where the human body is warned that it is closed to a limit position.

The active moments defined by relation (1) are:

$$M_{11} = \begin{cases} 0.08\,[Nm] & \text{if } 0.7\,[s] \le t \le 0.78\,[s] \\ -0.27\,[Nm] & \text{if } 0.89\,[s] \le t \le 1.16\,[s] \\ 0 & \text{otherwise} \end{cases} \tag{19}$$

$$M_{11} = \begin{cases} 0.08\,[Nm] & \text{if } 0.14\,[s] \le t \le 0.22\,[s] \\ -0.27\,[Nm] & \text{if } 0.92\,[s] \le t \le 1.19\,[s] \\ 0 & \text{otherwise} \end{cases} \tag{20}$$

These expressions for the active moments are approximations for the real situation. The experimental data are too little and they depend of a lot of factors. For a single gymnast, every experiment can lead to another set of data. Keeping account of these, we must work only with average of the experimental data.

In the next figures we captured the variation of the motion parameters versus time.

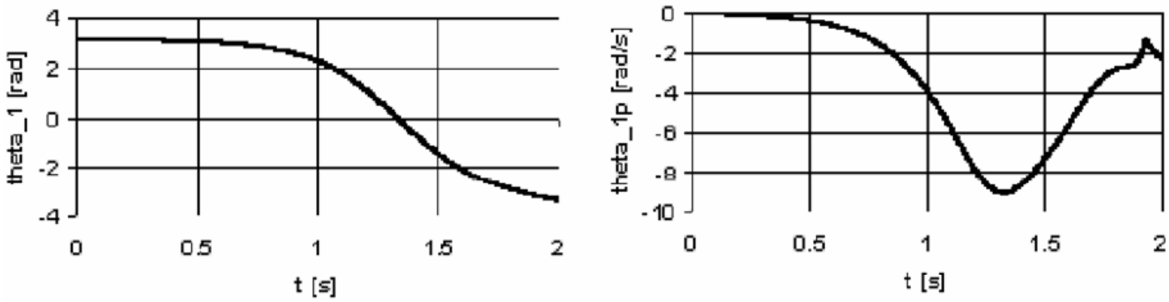

Fig. 3. Variations $\theta_1 = \theta_1(t)$ and $\dot{\theta}_1 = \dot{\theta}_1(t)$ for $0 \le t \le 2\,[s]$

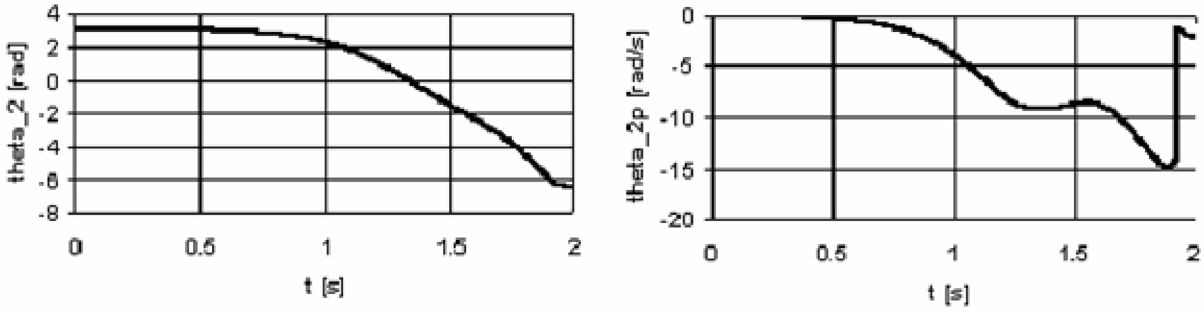

Fig. 4. Variations $\theta_2 = \theta_2(t)$ and $\dot{\theta}_2 = \dot{\theta}_2(t)$ for $0 \le t \le 2\,[s]$

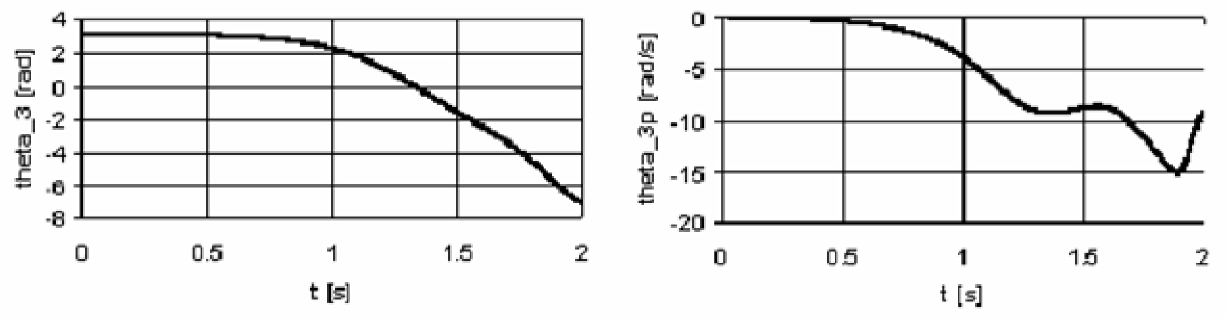

Fig. 5. Variations $\theta_3 = \theta_3(t)$ and $\dot{\theta}_3 = \dot{\theta}_3(t)$ for $0 \le t \le 2[s]$

4. CONCLUSIONS

We presented in this paper a simplified model for a gymnast. It is easy to observe that the mathematical equations are two complexes even in this situation. A more detailed model, considering more than three elements for the gymnast, can be made. Validation of the model consists in the comparison of the results of the simulation with the practical results.

The research in this field has just begun in the last years and there are not many available data. For this reason, a more complex model is not necessary now.

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