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THE EFFECT OF CYCLING VARYING MESH STTIFFNESS ON DYNAMIC MOTION CHARACTERISTICS OF SPUR GEARS

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Abstract. The paper presents a computer analysis of the interactions between time-varying mesh stiffness and dynamic motion characteristics of spur gear pairs. An accurate model and the variation of the gear mesh stiffness as a function of contact point along the line of contact are presented. The variation of angular velocity of the pinion and gear is presented in relation with the cyclic tooth mesh stiffness The effects of the tooth tip relief on the transmission error are included in the analysis.

1.Introduction

The dynamic characteristics of spur gear pairs are significant for the design and motion control of these mechanisms [3], [6], [9]. Reported studies of gear dynamics include Kahraman and Blankenship [5], Tamminana, Kahraman and Vijazakar [10], Yang and Sun [11]. The position accuracy in a motion control system is affected by vibration due to nonlinear effects such mesh stiffness or friction [4], [7], [8]. The angular speed fluctuation of the meshing gears and the dynamic transmission error are accepted as relevant parameters in dynamic analysis. The time-varying mesh stiffness represents the main cause of undesired vibrations in the case of gear transmissions with high manufacturing precision.

An investigation of the influence of the mesh stiffness on the dynamic motion characteristics of spur gear pairs is presented in the paper. In order to obtain reliable data for the prediction of gear dynamic behaviour, the dynamic model accounts the non-linear time varying mesh stiffness and tooth profile modifications.

2. Dynamic model

The mechanical model for a gear pair in mesh is shown in Figure 1. In this model, the teeth are considered as springs and the gear blanks as inertia masses. The differential equations of motion can be expressed as

$$J_1 \cdot \ddot{\theta}_1 + F_d \cdot r_{b1} = M_{t1} \tag{1}$$

$$J_2 \cdot \ddot{\theta}_2 - F_d \cdot r_{b2} = M_{t2} \tag{2}$$

where θ_1, θ_2 are the rotation angle of the pinion and the driven gear, respectively. J_1 and J_2 are the mass moments of inertia of the gears, M_{t1} and M_{t2} denote the external torques applied on the gear system.

The dynamic load is expressed as

$$F_d = \sum_{i=1}^{N} F_{di}(t) \tag{3}$$

where

$$F_{di} = k_i(t) \cdot [r_{b1} \cdot \theta_1 - r_{b2} \cdot \theta_2 + e_i(t)] + c \cdot \left(r_{b1} \cdot \dot{\theta}_1 - r_{b2} \cdot \dot{\theta}_2\right). \tag{3}$$

By introducing the composite coordinate

$$x = r_{b1} \cdot \theta_1 - r_{b2} \cdot \theta_2 . \tag{4}$$

Eqs. (1) and (2) yield an equation of motion in the following form

$$m \cdot \ddot{x} + c \cdot \dot{x} + \sum_{i=1}^{N} F_{di}(t) = F_n \tag{5}$$

where

$$F_{di}(t) = k_i(t) \cdot [x_s + x_d + e_i(t)] . \tag{6}$$

and where x_d is the dynamic displacement and N represents the number of simultaneous tooth pairs in mesh. For a pair of contacting teeth *i*, the time-varying mesh stiffness $k_i(t)$ and the composite tooth profile error $e_i(t)$ act as parameter excitations.

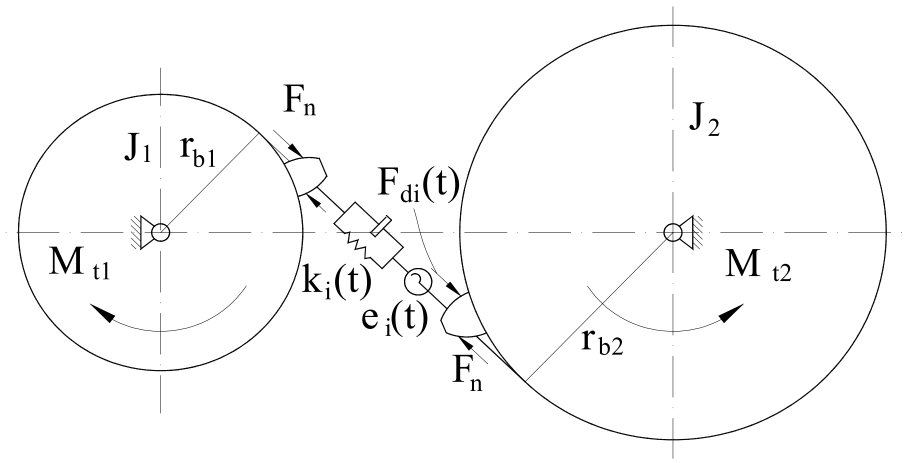

Fig.1. Dynamic model of gear pair

The static displacement x_s due to the static load F_n is calculated by

$$x_s = F_n / k_m$$

The meshing resonance frequency of the gear pair is determined as follows

$$f_n = \frac{1}{2 \cdot \pi} \cdot \sqrt{\frac{k_m}{m}} \tag{7}$$

where m represents the equivalent inertia mass and k_m is the average mesh stiffness of the gear pair.

3. MESH STIFFNESS

The gear tooth is modeled to be a nonuniform cantilever beam supported by a flexible fillet region and foundation [1] as shown in Figure 2. The total deflection f_j of a pair of meshing teeth is expressed as

$$f_j = \sum_{j=1}^{2} f_{bj} + \sum_{j=1}^{2} f_{fj} + f_H \tag{8}$$

where f_b - the deflection due to bending, shear and axial deformation of the tooth corresponding to the involute profile; f_f - the deflection due to the flexibility of the tooth foundation and fillet; f_H - the local compliance of the Hertzian contact.

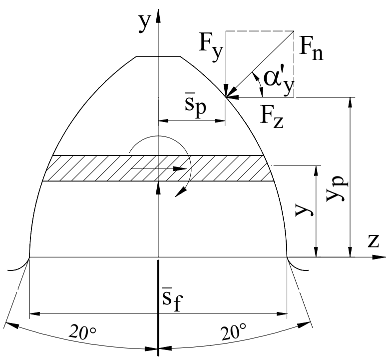

Fig.2. Gear tooth deflection model

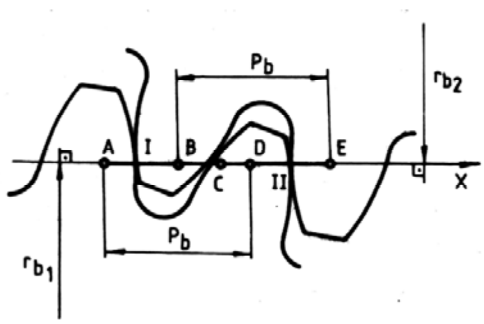

Fig.3. Specific contact points of mesh cycle

The individual tooth mesh stiffness is defined in the normal direction to the contact surface as

$$k_j = \frac{F_n}{f_j} \tag{9}$$

where F_n is the normal tooth load.

The teeth pairs in contact act like parallel springs. Therefore, the total mesh stiffness during each engagement cycle can be written as a function of the position of contact point on the action line

$$k_t = k_s^I + k_s^{II}, \quad \text{for double-tooth contact}$$

$$k_t = k_s^I, \quad \text{for simple - tooth contact}$$

where I and II are the mating points of the teeth pairs (Fig. 3).

Referring to Figure 3, the following mesh points were used to represent the successive positions of contact point of a tooth as it passes through the zone of loading: the initial point of engagement, A; the lowest point of single-tooth contact, B; the highest point of single-tooth contact, D; and the final point of engagement, E. Section AB and DE are double - tooth contact zone and section BD is the single tooth - contact zone.

The time-varying mesh stiffness is mainly caused by the following factors: (i) the variation of the single mesh stiffness along the line of action; (ii) the fluctuation of the total number of total pairs in contact during the engagement cycle.

Figures 4 and 5 show the variation of individual and total mesh stiffness from the starting to ending of contact in relation to the gear ratio. These examples illustrate the effect of the change of the number of meshing teeth pairs on the amount of the total mesh stiffness. For a gear pair, average mesh stiffness decreases with decreasing contact ratio.

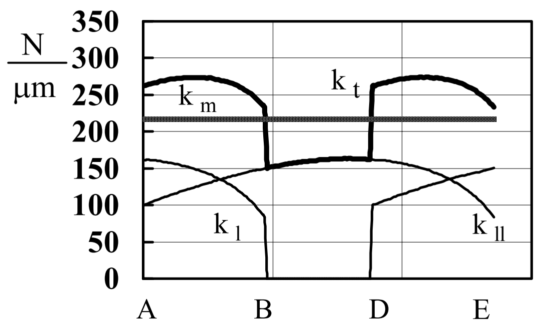

Fig.4. Variation of mesh stiffness components of gear pair GP1

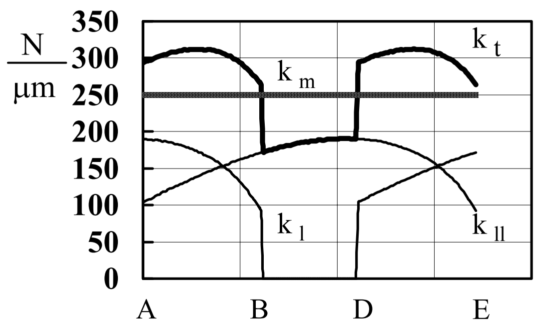

Fig.5. Variation of mesh stiffness components of gear pair GP2

4. Dynamic simulation

Specifications of geometrical and kinematics parameters of the analyzed gear pairs are shown in Table 1. These are for spur gear pairs having face-width of gears, b=12 [mm] and center distance, a = 70 [mm]. Additionally, the design parameters are chosen as: pinion speed, n_1 = 2800 [rpm]; damping ratio, ξ=0.12.

Table 1

Gear pair	z_1	z_2	m	x_1	x_2	ε_α	k_m [N/μm]	f_n [Hz]
GP1	18	29	3	0.5	-0.66	1.54	216.4	4419
GP2	18	76	1.5	0.5	-0.82	1.56	249.8	3591

In the analysis of dynamic loads, the transmitting load is defined as

$$W = q \cdot (F_n / b) \tag{10}$$

where q represents the load factor. A numerical value F_n / b = 80 N/mm corresponding to a medium transmitting load is considered in the numerical analysis.

A computer program was developed for simulating the dynamic characteristics of spur gear pairs. The equations of motion are solved by the fourth-order Runge-Kutta method. Computer analysis of dynamic characteristics includes different gear pairs with combination of the gear ratio and the amount of linear tip relief.

The variation of angular velocity of the pinion and gear are shown in Figs. 6 and 7, respectively. The fluctuation of the angular velocity in gear 2 is smaller than those of the pinion due to the larger moment of inertia of the gear 2 and the gear ratio because the inertia of the gear is larger than that the pinion.

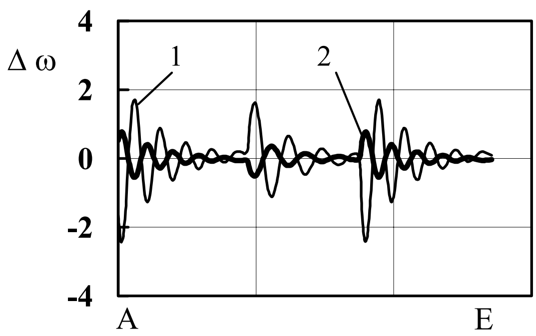

Fig. 6. Variation of angular velocities for gear pair GP1

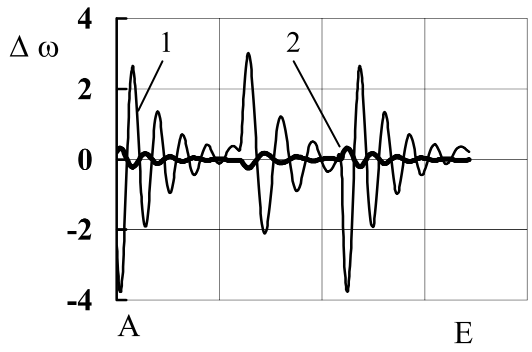

Fig. 7. Variation of angular velocities for gear pair GP2

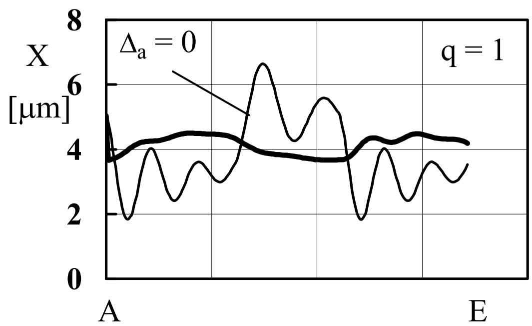

Fig. 8. Variation of dynamic transmission error of gear pair GP1

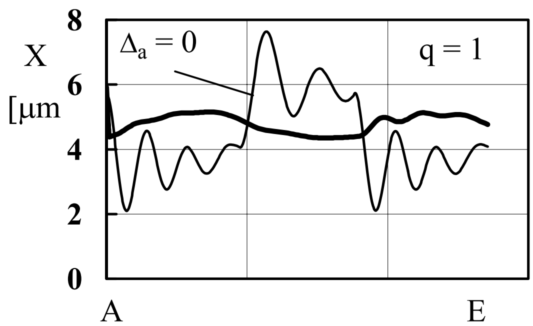

Fig. 9. Variation of dynamic transmission of gear pair GP2

For gear pairs with addendum modifications, the amount of the tip relief applied at the tooth tip of pinion is equal to the teeth deflection evaluated at the highest point of single-tooth contact. For gear, the tip relief amount is equal to the teeth deflection at the lowest point of single-tooth contact. The length of the tooth tip relief depends upon the transverse contact ratio ε_α and is calculated by

$$l_a = (\varepsilon_\alpha - 1) \cdot p_b \tag{10}$$

where p_b is the base pitch.

The effects of the involute tip relief on the variation of the dynamic transmission error of spur gear pairs are presented in Figures 8 and 9. The linear tip relief is introduced according to the tooth deformation for a nominal specific load F_n / b = 80 N/mm .

5. Conclusions

An analytical procedure for calculating dynamic characteristics of spur gear pairs with tooth profile modifications is presented. This procedure predicts the effect of tooth tip relief on the dynamic factor and transmission error variation during the meshing cycle. The time-varying mesh stiffness along the path of contact is found by using an exact analytical model. Gear pairs having a constant magnitude tip relief of varying extent are analyzed over a range of specific load conditions. The following observations should be noted:

(1) The mesh stiffness variation is a function of tooth contact position along the line of action.

(2) The influence of gear ratio on dynamic motion charactersitics of spur gear pairs was examined by using the mesh stiffness parameter.

(3) A computer programs to analyze the effect of linear involute tip relief on dynamic characteristics of spur gear pairs was developed.

(4) An optimum involute tip relief can be predicted to ensure minimum dynamic transmission error for a particular design force.

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