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KINETOSTATIC CALCULUS FOR PUMPING UNITS WITH REVERSE SCHEME

Liliana Rusu

Petroleum – Gas University, Ploiesti, e-mail:lianarusu8@yahoo.com

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Abstract: This paper presents a method of kinetostatic calculus for pumping units with reverse scheme which determines the bearings reactions and also the motor moment that the reducer applies on the crank. The computer program written in Mathcad 2001 having this mathematic model , as base, is flexible, allows any input parameter and throws exact outputs .

1.Theoretical premises

The elements of the quadrangle mechanism that drives the rod pumping unit with reverse scheme, are the ones described in Fig. 1:

- counter balanced crank adjusted by an angle α , 1;
- connecting rod, 2;
- walking beam, 3.

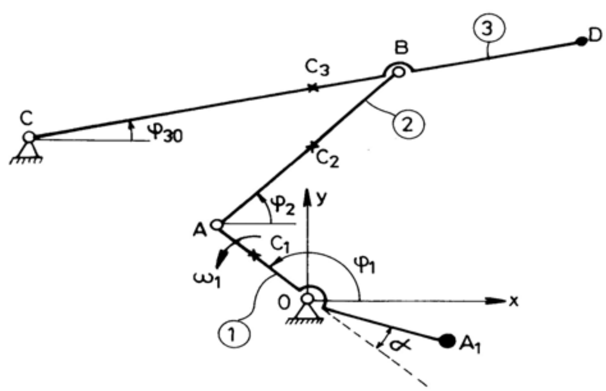

Fig.1 The mechanism of the reverse scheme pumping unit

The following steps have to be performed:

- ***Reduction of massic forces in the fixed point of the axes***

The mentioned elements are being insulated and they are charged with massic forces (Fig. 2). The direction of forces and moments are given, considering their values as being positive for the chosen direction.

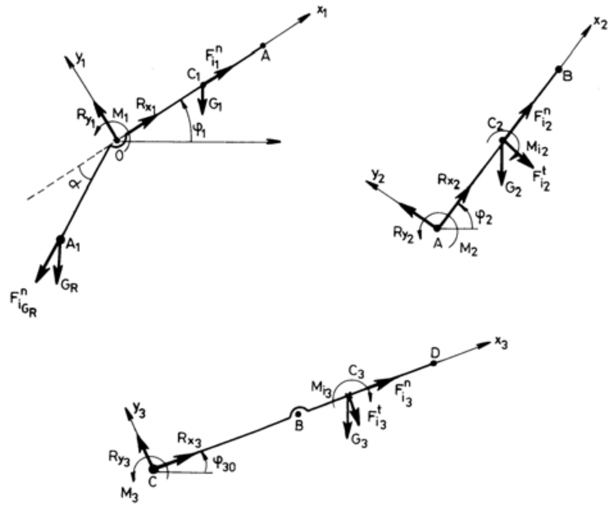

Fig. 2 Calculus schema for the reduction of massic forces

The relations of calculus for these forces and moments are provided in Tab. 1.

Tab 1

Item	Weight	Regular inertia force	Tangential inertia force	Inertia moment
Crank	G_1	$F_{i1}^n = \frac{G_1}{g}\omega_1^2 \cdot OC_1$		
Rotating balance	G_R	$F_{iG_R}^n = \frac{G_R}{g}\omega_1^2 \cdot OA_1$		
Rod connecting	G_2	$F_{i2}^n = \frac{G_2}{g}\omega_2^2 \cdot AC_2$	$F_{i2}^t = \frac{G_2}{g}\varepsilon_2 \cdot AC_2$	$M_{i2} = J_2 \cdot \varepsilon_2$
Walking beam	G_3	$F_{i3}^n = \frac{G_3}{g}\omega_3^2 \cdot CC_3$	$F_{i3}^t = \frac{G_3}{g}\varepsilon_3 \cdot CC_3$	$M_{i3} = J_3 \cdot \varepsilon_3$

The components of the torsor of the massic forces in point O of the crank, in point A of the connecting rod and in point C of the walking beam are calculated using the relations:

$$T_o \begin{cases} R_{x1} = F_{i1}^n - F_{iG_r}^n \cos\alpha - (G_1 + G_R)\sin\varphi_1 \\ R_{y1} = -(G_1 + G_R)\cos\varphi_1 - F_{iG_R}^n \sin\alpha \\ M_1 = (-G_1\cos\varphi_1)\cdot OC_1 + [G_R\cos(\varphi_1 + \alpha)]OA_1 \end{cases} \quad (1)$$

$$T_B\begin{cases} R_{x2} = F_{i2}^{n} - G_2\sin\varphi_2 \\ R_{y2} = -F_{i2}^{t} - G_2\cos\varphi_2 \\ M_2 = -M_{i2} - \left(F_{i2}^{t} + G_2\cos\varphi_2\right)\cdot AC_2 \end{cases} \tag{2}$$

$$T_C\begin{cases} R_{x3} = F_{i3}^{n} - G_3\sin\varphi_{30} \\ R_{y3} = F_{i3}^{t} - G_3\cos\varphi_{30} \\ M_3 = -M_{i3} - \left(F_{i3}^{t} + G_3\cos\varphi_{30}\right)\cdot CC_3 \end{cases} \tag{3}$$

- ***Kinetostatic equilibrium equation***

After presenting the kinetostatic equilibrium equations for the three elements, we will solve the bearing reactions and also the motor moment applied by the reducer to the crank. In Fig. 3, there are presented, besides the reducing massic forces components, the reactions in joints O, A, B and C drew in the positive direction of local axes and the technological force F.

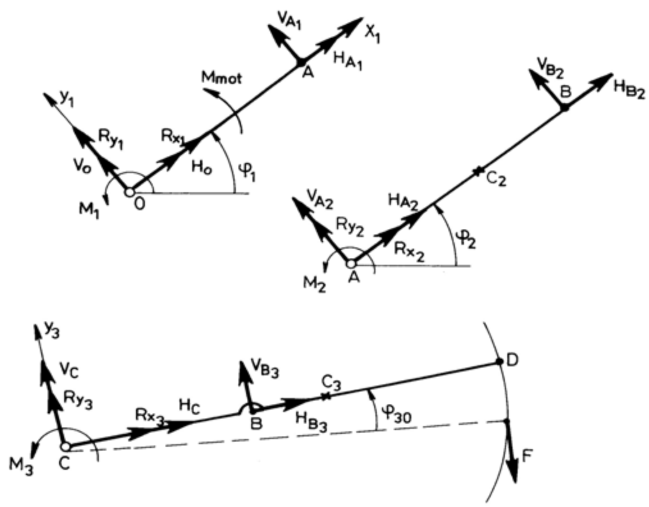

Fig. 3 Kinetostatic calculus schema

The equations for kinetostatic equilibrium are:

$$\begin{cases} H_{B3} + H_C + R_{x3} - F\sin\varphi_{30} = 0 \\ V_{B3} + V_C + R_{y3} - F\cos\varphi_{30} = 0 \\ M_3 + V_{B3} \cdot CB - F \cdot CE = 0 \\ H_{A2} + H_{B2} + R_{x2} = 0 \\ V_{A2} + V_{B2} + R_{y2} = 0 \\ M_2 + V_{B2} \cdot AB = 0 \\ H_O + H_{A1} + R_{x1} = 0 \\ V_O + V_{A1} + R_{y1} = 0 \\ M_{mot} + M_1 + V_{A1} \cdot OA = 0 \end{cases} \tag{4}$$

To these relations we add the connection equations between the reactions applied to each Element of the same joint (B joint and A joint):

$$\begin{cases} H_{B3}\cos\varphi_{30} - V_{B3}\sin\varphi_{30} = -(H_{B2}\cos\varphi_2 - V_{B2}\sin\varphi_2) \\ H_{B3}\sin\varphi_{30} + V_{B3}\cos\varphi_{30} = -(H_{B2}\sin\varphi_2 + V_{B2}\cos\varphi_2) \end{cases} \tag{5}$$

$$\begin{cases} H_{A2}\cos\varphi_2 - V_{A2}\sin\varphi_2 = -(H_{A1}\cos\varphi_1 - V_{A1}\sin\varphi_1) \\ H_{A2}\sin\varphi_2 + V_{A2}\cos\varphi_2 = -(H_{A1}\sin\varphi_1 + V_{A1}\cos\varphi_1) \end{cases} \tag{6}$$

- ***Reactions and motor moment calculus***

From (1 - 6), we get the calculus relations for the reactions that take place in joints O, A and B, i.e R_o , R_a , R_b , R_c for motor moment M_{mot}:

$$\begin{aligned} & V_{B3} = (F \cdot CE - M_3)/CB; V_C = F\cos\varphi_{30} - R_{y3} - V_{B3} \\ & V_{B2} = -M_2/AB; V_{A2} = -R_{y2} - V_{B2} \\ & H_{B3} = [V_{B2} + V_{B3}\cos(\varphi_2 - \varphi_{30})]/\sin(\varphi_2 - \varphi_{30}) \\ & H_{B2} = V_{B_2}\cos(\varphi_{30} - \varphi_2) + V_{B3}/\sin(\varphi_{30} - \varphi_2) \\ & H_C = F\sin\varphi_{30} - R_{x3} - H_{B3}; H_{A2} = -R_{x2} - H_{B2} \\ & H_{A1} = V_{A2}\sin(\varphi_2 - \varphi_1) - H_{A2}\cos(\varphi_2 - \varphi_1) \\ & V_{A1} = -V_{A2}\cos(\varphi_2 - \varphi_1) - H_{A2}\sin(\varphi_2 - \varphi_1) \\ & H_O = -R_{x1} - H_{A1}; V_O = -R_{y1} - V_{A1} \\ & M_{mot} = -M_1 - V_{A1} \cdot OA \end{aligned} \tag{7}$$

The reactions in joints O, A , B and C are calculated using the relations:

$$R_0 = \sqrt{H_0^2 + V_0^2} \tag{8}$$

$$R_A = \sqrt{H_{A1}^2 + V_{A1}^2} = \sqrt{H_{A2}^2 + V_{A2}^2} \tag{9}$$

$$R_B = \sqrt{H_{B2}^2 + V_{B2}^2} = \sqrt{H_{B3}^2 + V_{B3}^2} \tag{10}$$

$$R_C = \sqrt{H_C^2 + V_C^2} \tag{11}$$

2.Calculus example (for a reverse scheme pumping unit)

There are presented the variation graphics of the above calculated reactions, in fig. 5 – 8, for a reverse scheme rod pumping unit (API Spec. 11E, class III - VULCAN) for which the kinematic was studied in [4] and for which the balance is done using a rotative weight G_r=70000N at a distance OA_1=2,55m, adjusted by an angle $\alpha = 0{,}4\text{rad}$.
We shall use the dynagraph simplified in 6 points.
The variation of the technological force F, according to angle φ_1, is presented in Fig. 4.

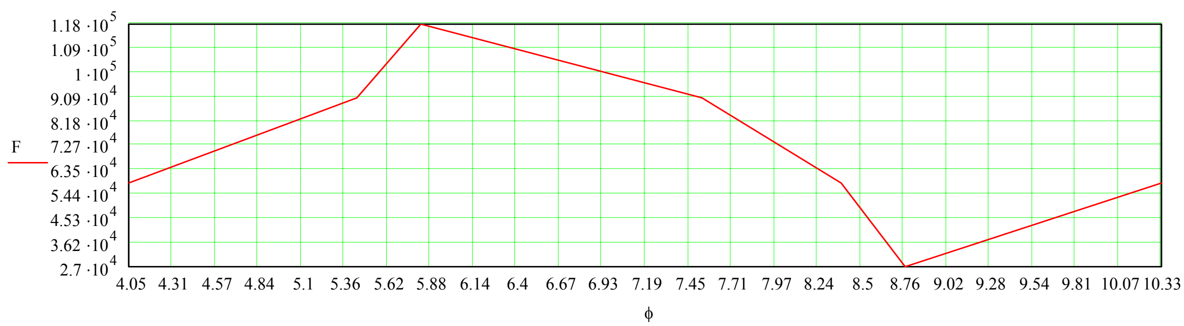

Fig.4 The graphic of the technological force F according to angle φ_1

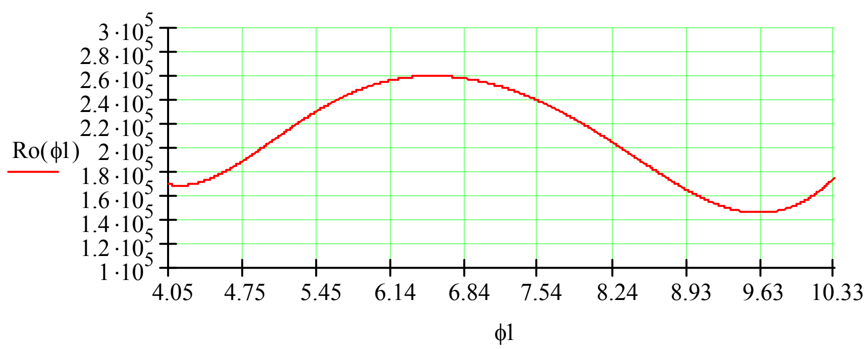

Fig.5 The variation graphic of the reaction in O joint

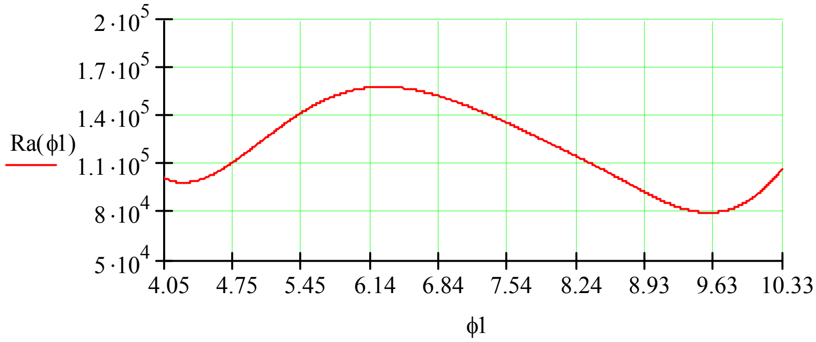

Fig. 4 The variation graphic of the reaction in A joint

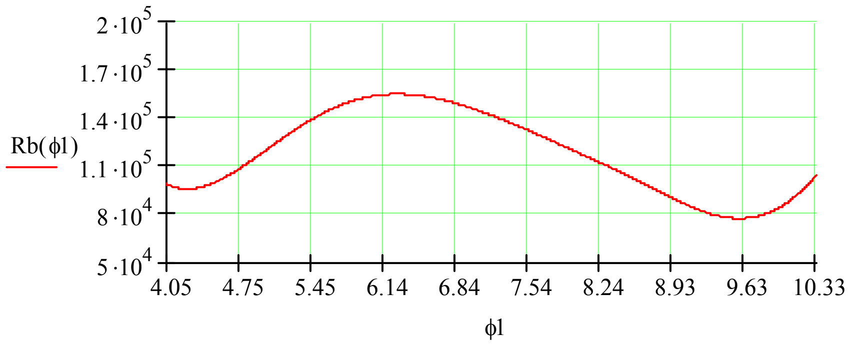

Fig. 5 The variation graphic of the reaction in B joint

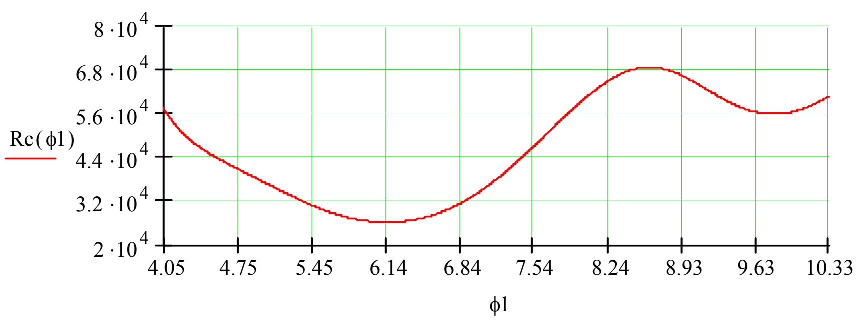

Fig. 6 The variation graphic of the reaction in C joint

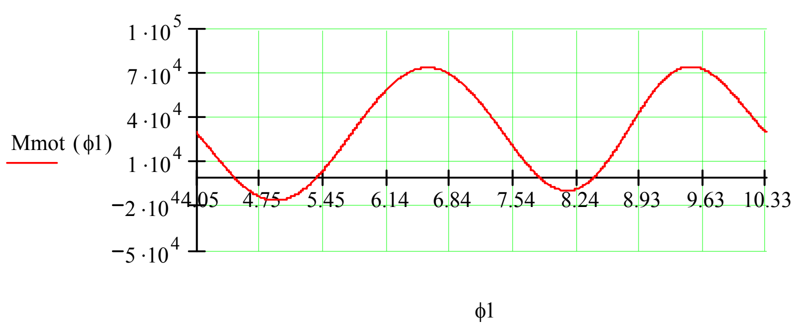

Fig. 7 The variation graphic of the motor moment

3.Conclusions

The suggested method leads to a rapid calculus of the reactions in the kinematic couplings of the pumping unit with reverse scheme and also of the motor moment of the gear reducer.
The computer program written in Mathcad 2001 having this mathematic model, as base, is flexible, allows any input parameter and throws exact outputs that can be used by pumping unit designers.

4.Bibliography

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