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DYNAMIC MODELLING OF N-CARDAN TRANSMISSIONS WITH SHAFTS IN A SPATIAL CONFIGURATION. Part I. TRANSMISSIONS' MODULARIZATION

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Abstract: In the first part of this paper there are synthetically presented the structural typical modules met in n-Cardan transmissions and there are established the generalized analytical models of the forces from these modules. By means of these modules, the dynamic computational modelling of these transmissions can be generalized and become more simple and unitary. Taking into account this modularizing, an algorithm for the dynamic modelling is presented in the second part of the paper, base don an example of 2- Cardan transmission.

1. INTRODUCTION

In the modern computer techniques conditions, an important role in developing performant software belongs to the simplification of algorithms through modularization. In this context, the paper presents the modular modelling of the dynamics of n-Cardan transmissions in spatial configuration.

In Fig. 1 there are represented the representative schemes of statically determined n-Cardan transmissions with *two* couplings (Fig. 1,a), with *three* couplings (Fig. 1,b and c) and with *four* couplings (Fig. 1.d). In accordance with these schemes, two distinct classes of component modules can be distinguished: *element* modules and *constraint* modules. The *element*-modules, which are typical to the n-Cardan transmissions, are referring to: 1° the Cardan cross, 2° the Cardan joint fork, 3° the extremal shaft, 4° the intermediary unsupported shaft, 5° the intermediary supported shaft. Due to Fig. 1, the *constraint*-modules are connections like the following: 1° Cardan fork-cardan cross, 2° base- extremal Cardan shaft, 3° base-supported intermediary Cardan shaft.

The modelling of the forces from a classical n-Cardan transmission can be reduced to the establishment of the correlations between the forces of each element module, considered in general. Therefore, the correlations between the forces of each element module and each constraint module will be presented further on, taking into account the following notations and prerequisites.

2. NOTATIONS AND PREMISES

2r — the distance between the contact zones of a Cardan fork (Fig. 2);

$O_i u_{i1} g_{i1} n_{i1}$, $O_i u_{i1} u_{i2} z$ and $O_i u_{i2} g_{i2} n_{i2}$ — reference frames, fixed to the input fork (1), to the Cardan cross and, respectively, to the output fork (2) from the i Cardan joint –see Fig. 2;

T_{i1}, B_{i1}, K_{i1}; T_{i2}, B_{i2}, K_{i2} — the modules of the torsional moment, bending moment and of the resultant moment which load the input forks (1) and, respectively, the output forks (2), from the i Cardan joint (see Fig. 2);

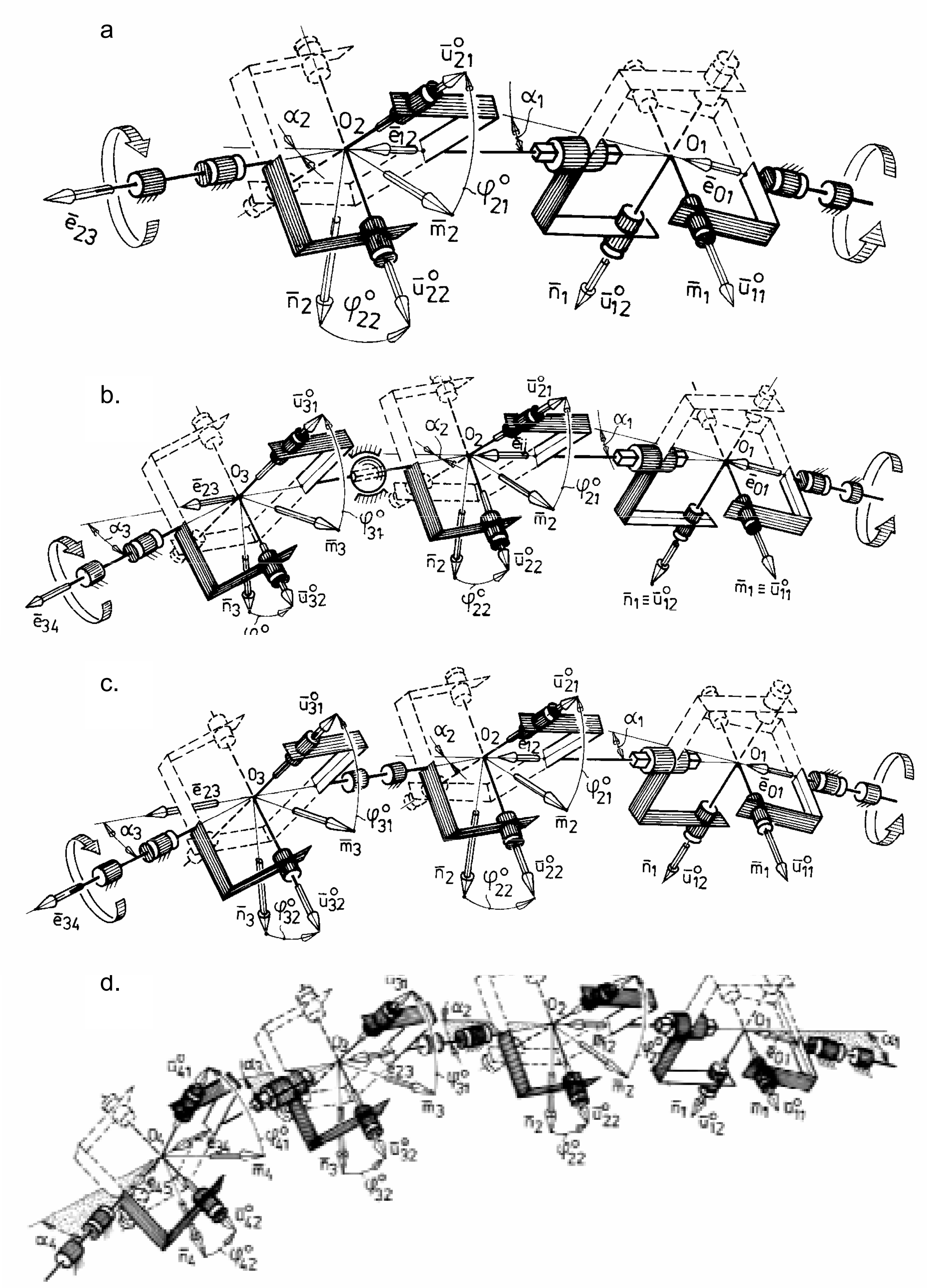


Fig. 1. *Variants of statically determined n-Cardan transmissions*

M_{11}, M_{n2} — the module of the motor moment and, respectively, of the resistant moment, from a n-Cardan transmission (Fig. 6);

A_{i1}, R_{i1}, H_{i1}; A_{i2}, R_{i2}, H_{i2} — the module of the axial force ($\overline{A}_i \parallel \overline{g}_i$), radial force ($\overline{R}_i \parallel \overline{u}_i$) and of the orthogonal force ($\overline{H}_i \parallel \overline{n}_i$) that load the input forks (1) and the output forks (2), in the *i* Cardan joint (see Fig. 2);

F_μ — the module of the friction force from the telescopic connection of the intermediary unsupported shaft (see Fig. 5);

P_{i1}, U_{i1}, V_{i1}, W_{i1}; P_{i2}, U_{i2}, V_{i2}, W_{i2} — the modules of the radial reactions from the supports of the extremal shafts (see Fig. 6);

S_{11}, S_{ij}, S_{n2} — the modules of the axial reactions from the supports of the extremal shafts (see Fig. 6);

a_{11}, b_{11}; a_{n2}, b_{n2} — the distances which define the positions of the radial reactions from the input shafts supports, and respectively, output shafts supports (see Fig. 6);

$F_{i1}^{'}$, $F_{i1}^{''}$, $N_{i1}^{'}$, $N_{i1}^{''}$, Q_{i1}; $F_{i2}^{'}$, $F_{i2}^{''}$, $N_{i2}^{'}$, $N_{i2}^{''}$, Q_{i2} — the modules of the reactions that are characteristic to the Cardan crosses arms, which are adjacent to the input fork (1) and, respectively, output fork (2), in the i Cardan joint (see Fig. 3);

$A_{i1}^{'}$, $A_{i1}^{''}$, $H_{i1}^{'}$, $H_{i1}^{''}$, R_{i1}; $A_{i2}^{'}$, $A_{i2}^{''}$, $H_{i2}^{'}$, $H_{i2}^{''}$, R_{i2} — the modules of the reactions that are characteristic to the input forks (1) and, respectively, output forks (2), from the i Cardan joint (see Fig. 4);

The dynamic modelling is made under the premise that the elements are rigid bodies.

3. MODELLING OF THE FORCES FROM THE CROSS-FORK CONNECTION

Due to Fig. 2,a, the following correlations can be written:

$$\left\{ T_{i1,2} = J_{i1,2} \cdot \varepsilon_{i1,2} - M_{i1,2}\,; B_{i1,2} = T_{i1,2} \cdot \mathrm{tg}\,\theta_{i1,2}\,; K_{i1,2} = \frac{T_{i1,2}}{\cos\theta_{i1,2}}, \right. \tag{1}$$

in which J and ε are the moment of inertia and respectively the angular acceleration.

According to Fig. 2,b, the forces with which the Cardan cross loads the two conjugate forks, have the following expressions:

$$\begin{cases} A_{i1,2} = A_{i2,1} \cdot \cos\theta_{i1} \cdot \cos\theta_{i2} + H_{i2,1} \cdot \cos\theta_{i1,2} \cdot \sin\theta_{i2,1} + R_{i2,1} \cdot \sin\theta_{i1,2}\,, \\ H_{i1,2} = A_{i2,1} \cdot \sin\theta_{i1,2} \cdot \cos\theta_{i2,1} + H_{i2,1} \cdot \sin\theta_{i1,2} \cdot \sin\theta_{i2,1} - R_{i2,1} \cdot \cos\theta_{i1,2}\,, \\ R_{i1,2} = A_{i2,1} \cdot \sin\theta_{i2,1} - H_{i2,1} \cdot \cos\theta_{i2,1}\,. \end{cases} \tag{2}$$

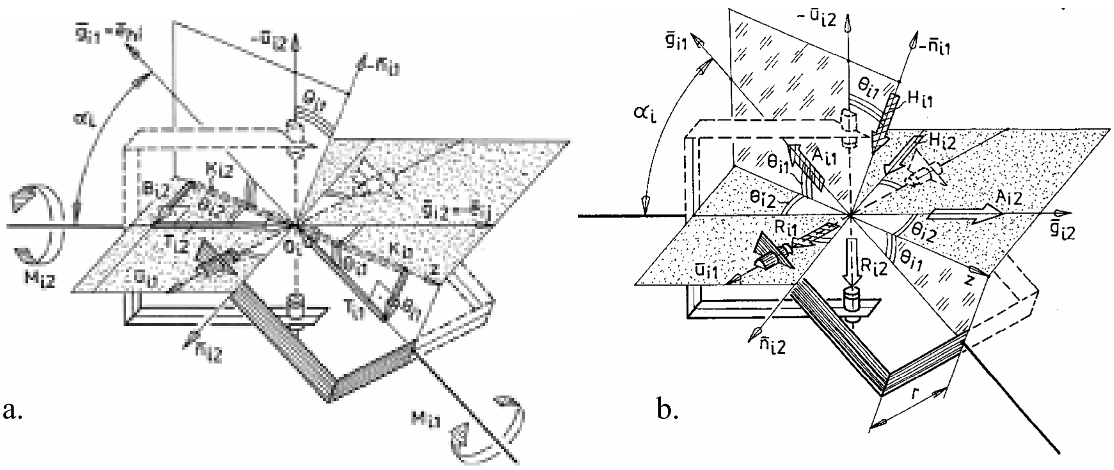


Fig. 2. *Constraints of cross-fork type*

4. THE MODULE OF THE CORRELATIONS BETWEEN A CARDAN CROSS FORCES

From Fig. 3 can be deduced the following dependencies (see also Fig. 4), which form the module of the correlations between the forces of a cardanic cross:

$$\begin{cases} Q_{i1,2} = -R_{i1,2}\,, \\ F'_{i1,2} = 0{,}5\cdot\left(-H_{i1,2}\cdot\cos\theta_{i1,2} + A_{i1,2}\cdot\sin\theta_{i1,2} + K_{i1,2}/r\right), \\ F''_{i1,2} = 0{,}5\cdot\left(-H_{i1,2}\cdot\cos\theta_{i1,2} + A_{i1,2}\cdot\sin\theta_{i1,2} - K_{i1,2}/r\right), \\ N'_{i1,2} = N''_{i1,2} = \pm 0{,}5\cdot\left(H_{i1,2}\cdot\sin\theta_{i1,2} + A_{i1,2}\cdot\cos\theta_{i1,2}\right). \end{cases} \tag{3}$$

The superior sign from the final relation (3) corresponds to the input arm of the cross (coefficient 1), while the inferior sign corresponds to the output arm (coefficient 2).

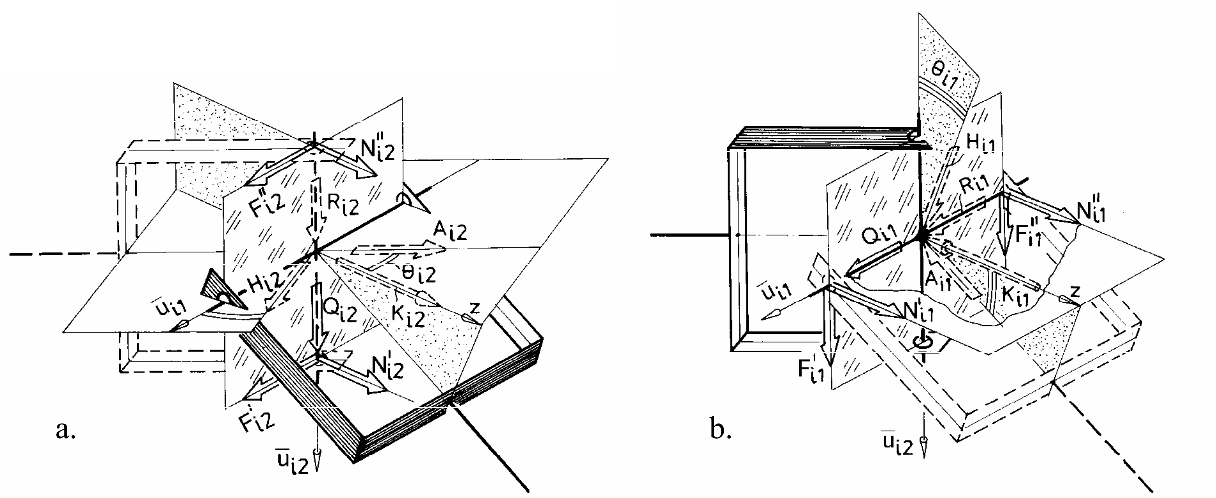


Fig. 3. *The forces of a Cardan cross*

5. THE MODULE OF THE CORRELATIONS BETWEEN THE CARDAN FORK FORCES

The forces with which the cross acts on a fork are represented in Fig. 4 with a *continuous line*, while the forces with which the fork acts on the cross are represented with a *dashed line*; the following correlations interfere between these forces:

$$\begin{cases} R'_{i1,2} = R_{i1,2}\,, A'_{i1,2} = \dfrac{1}{2}\cdot\left(A_{i1,2} + \dfrac{B_{i1,2}}{r}\right), A''_{i1,2} = \dfrac{1}{2}\cdot\left(A_{i1,2} - \dfrac{B_{i1,2}}{r}\right), \\ H'_{i1,2} = \dfrac{1}{2}\cdot\left(H_{i1,2} - \dfrac{T_{i1,2}}{r}\right), \qquad H''_{i1,2} = \dfrac{1}{2}\cdot\left(H_{i1,2} + \dfrac{T_{i1,2}}{r}\right). \end{cases} \tag{4}$$

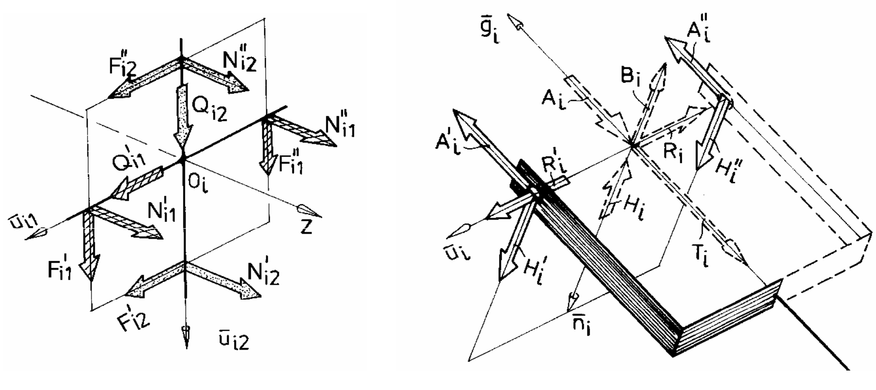


Fig. 4. *The forces which load the Cardan cross*

6. THE MODULE OF THE CORRELATIONS BETWEEN THE FORCES OF A INTERMEDIARY SHAFT

6.1 *The unsupported intermediary shaft*

The general case of a *intermediary unsupporte*d Cardan shaft in a telescopic variant is presented in Fig. 5.

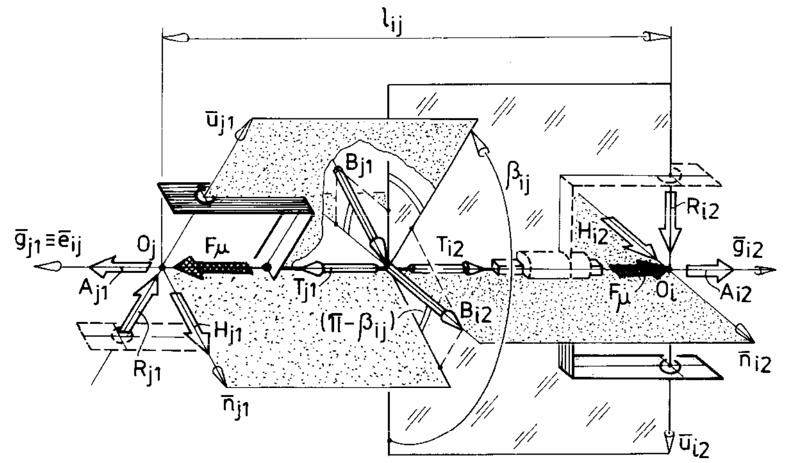


Fig. 5. *The forces of an unsupported telescopic intermediary shaft*

The following dependencies are obtained in the case of the shaft from Fig. 5:

$$\begin{cases} T_{i2} = T_{j1} + J_{ij} \cdot \varepsilon_{ij}, A_{i2} = A_{j1} = -F_{\mu ij}, R_{i2} = \dfrac{B_{i2} - B_{j1} \cdot \cos \beta_{ij}}{l_{ij}}, \\ H_{i2} = -\dfrac{B_{j1} \cdot \sin \beta_{ij}}{l_{ij}}, R_{j1} = \dfrac{B_{j1} - B_{i2} \cdot \cos \beta_{ij}}{l_{ij}}, H_{j1} = -\dfrac{B_{i2} \cdot \sin \beta_{ij}}{l_{ij}}. \end{cases} \quad (5)$$

6.2 *Intermediary* shaft *supported by a 4 DOF polyjoint*

The intermediary Cardan shaft that is supported on the base by a 4 DOF polyjoint *O* (Fig. 6), is typical for the statically determined transmissions with an odd number of couplings (see also Fig. 1,b), in which the *cross-fork* constraints are *revolute* polyjoint (a usual case for the technical applications!).

The general scheme for the forces and moments of this intermediary shaft is represented in Fig. 6.

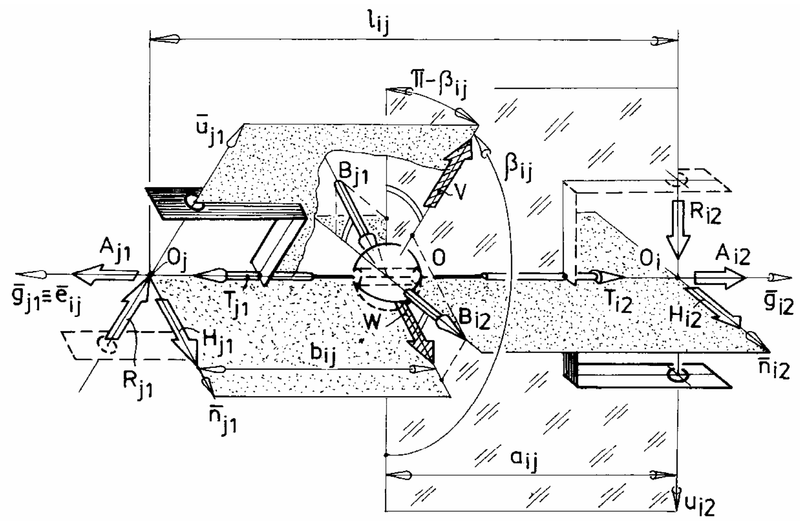


Fig. 6. *The forces of an intermediary shaft supported by a 4 DOF polyjoint*

The following correlations can be written by means of Fig. 6 and in the premise of knowing the forces of i_2 fork:

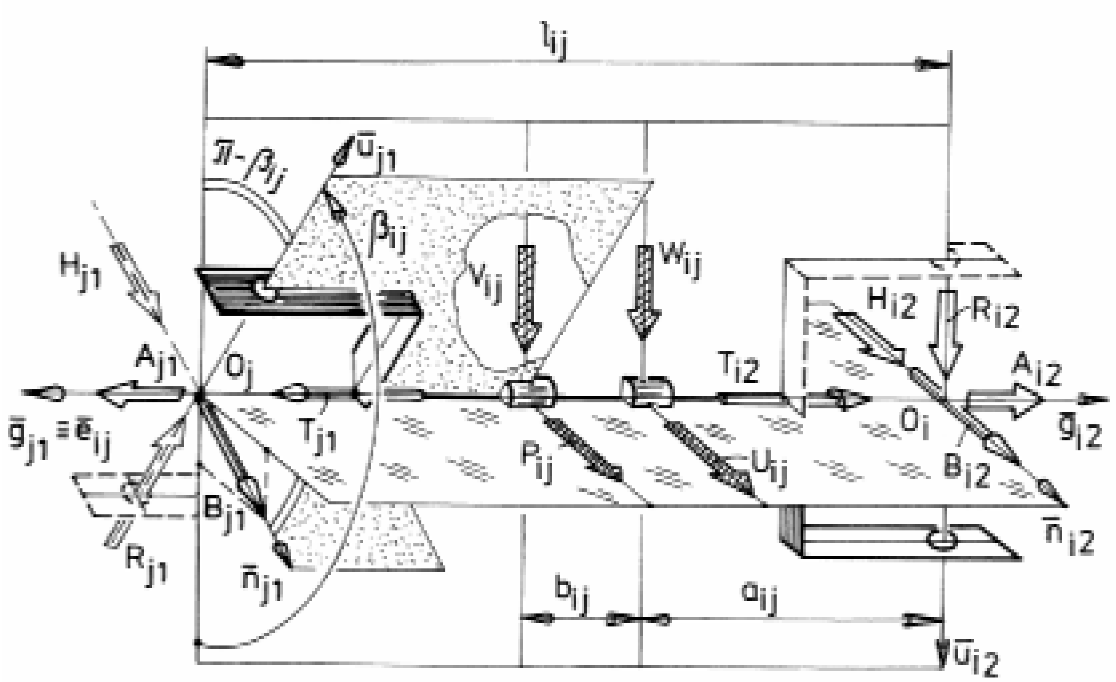
$$\begin{cases} T_{j1} = T_{i2} - J_{ij} \cdot \varepsilon_{ij} , \\ A_{j1} = A_{i2} , \\ V_{ij} = \dfrac{B_{i2} \cdot cos\,\beta_{ij} - B_{j1} - \left(R_{i2} \cdot cos\,\beta_{ij} + H_{i2} \cdot sin\,\beta_{ij}\right) \cdot l_{ij}}{b_{ij}} , \\ W_{ij} = \dfrac{B_{i2} \cdot sin\,\beta_{ij} - \left(R_{i2} \cdot sin\,\beta_{ij} - H_{i2} \cdot cos\,\beta_{ij}\right) \cdot l_{ij}}{b_{ij}} , \\ R_{j1} = \dfrac{- B_{i2} \cdot cos\,\beta_{ij} + B_{j1} + \left(R_{i2} \cdot cos\,\beta_{ij} + H_{i2} \cdot sin\,\beta_{ij}\right) \cdot a_{ij}}{b_{ij}} , \\ H_{j1} = \dfrac{- B_{i2} \cdot sin\,\beta_{ij} + \left(R_{i2} \cdot sin\,\beta_{ij} - H_{i2} \cdot cos\,\beta_{ij}\right) \cdot a_{ij}}{b_{ij}} . \end{cases} \tag{6}$$

6.3 *Intermediary shaft supported by a revolute or cylindrical polyjoint*

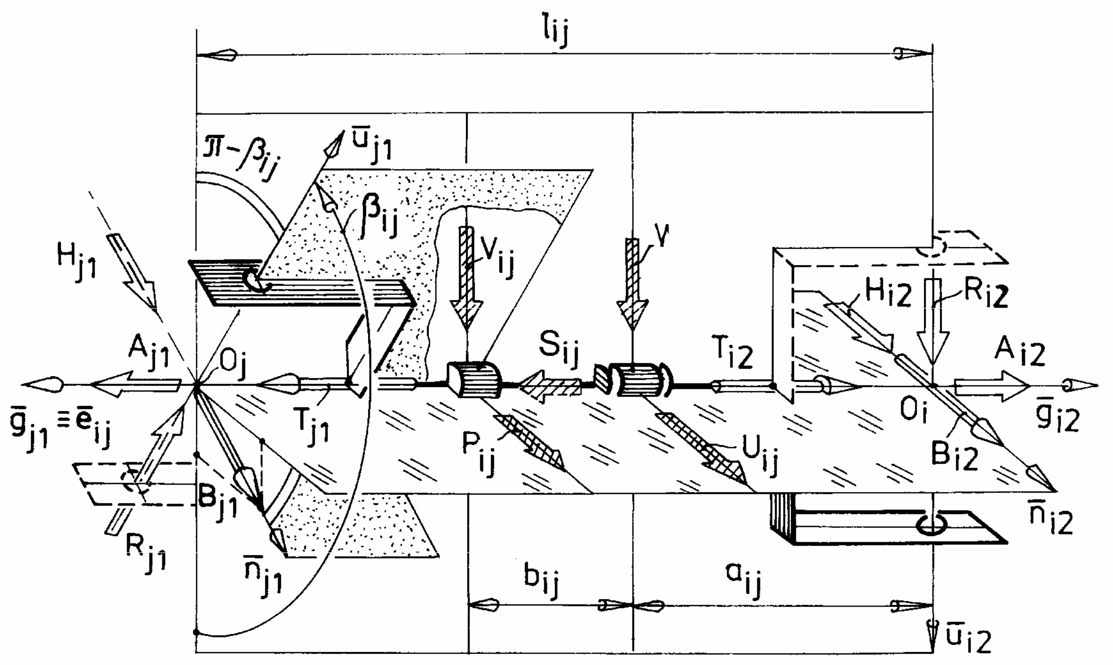
The intermediary Cardan shaft that is supported on the base by a revolute joint (1 DOF) usually links two adjacent 2-Cardan transmissions with telescopic intermediary shafts (see Fig. 1,d).

The general scheme, with the forces and moments of this type of shaft, is illustrated in Fig. 7, a.

The intermediary shaft that is supported by a *cylindrical polyjoint* (2 DOF) can be derived from the previous case (Fig. 7,a), by eliminating the collars of the revolute joint; in other words, this shaft forces model (Fig. 7,b) can be obtained from the previous shaft model (Fig. 7,a), by nullifying the axial force S_{ij}.



a.



b.

Fig. 7. *The forces of an intermediary shaft supported by a revolute or cylindrical polyjoint*

This type of Cardan shaft (Fig. 7,b) is used in the following cases:

a) in statically determined transmissions with an odd number of couplings, in which the cross-forks constraints, from the *j* coupling are materialized through cylindrical polyjoint (see Fig. 1,c) and

b) in transmissions with an even number of couplings, in which intermediary unsupported and non-telescopic shafts are also used; thus, if in the transmission from Fig. 1,d, one of the telescopic shaft becomes non-telescopic, then the transmission remains statically determined only if the revolute polyjoint of the median shaft becomes cylindrical polyjoint.

The correlations specific to the forces from scheme a (Fig. 7) are further established and then, by particularization, the correlations specific to scheme b are established.

The following dependencies can be written based on the scheme from Fig. 7,a:

$$\begin{cases} S_{ij} = A_{i2} - A_{j1}\,, \qquad T_{j1} = T_{i2} - J_{ij}\cdot\varepsilon_{ij}\,, \\ P_{ij}\cdot b_{ij} + \left(l_{ij} - a_{ij}\right)\cdot\left[R_{j1}\cdot cos\left(\beta_{ij} - 90°\right) + H_{j1}\cdot sin\left(\beta_{ij} - 90°\right)\right] = \\ \qquad = H_{i2}\cdot a_{ij} + B_{j1}\cdot cos\left(\beta_{ij} - 90°\right), \\ U_{ij}\cdot b_{ij} + B_{j1}\cdot cos\left(\beta_{ij} - 90°\right) + H_{i2}\cdot\left(a_{ij} + b_{ij}\right) = \\ \qquad = \left(l_{ij} - a_{ij} - b_{ij}\right)\cdot\left[R_{j1}\cdot cos\left(\beta_{ij} - 90°\right) + H_{j1}\cdot sin\left(\beta_{ij} - 90°\right)\right], \\ V_{ij}\cdot b_{ij} + B_{i2} + B_{j1}\cdot sin\left(\beta_{ij} - 90°\right) + \left(l_{ij} - a_{ij}\right)\cdot H_{j1}\cdot cos\left(\beta_{ij} - 90°\right) = \\ \qquad = R_{i2}\cdot a_{ij} + \left(l_{ij} - a_{ij}\right)\cdot R_{j1}\cdot sin\left(\beta_{ij} - 90°\right), \\ W_{ij}\cdot b_{ij} + R_{i2}\cdot\left(a_{ij} + b_{ij}\right) + \left(l_{ij} - a_{ij} - b_{ij}\right)\cdot R_{j1}\cdot sin\left(\beta_{ij} - 90°\right) = \\ \qquad = B_{i2} + B_{j1}\cdot sin\left(\beta_{ij} - 90°\right) + \left(l_{ij} - a_{ij} - b_{ij}\right)\cdot H_{j1}\cdot cos\left(\beta_{ij} - 90°\right). \end{cases} \tag{7}$$

The module of the correlations, which are specific to the intermediary shaft that is supported by a revolute polyjoint (Fig. 7,a) is obtained from the previous relations; in a simplified form, it results (by not considering the coefficients *i* and *j*):

$$\begin{cases} S = A_2 - A_1\ , \\ T_1 = T_2 - J \cdot \varepsilon\ , \\ P = \dfrac{H_2 \cdot a + B_1 \cdot sin\,\beta - (l-a)\cdot (R_1 \cdot sin\,\beta - H_1 \cdot cos\,\beta)}{b}\ , \end{cases}$$

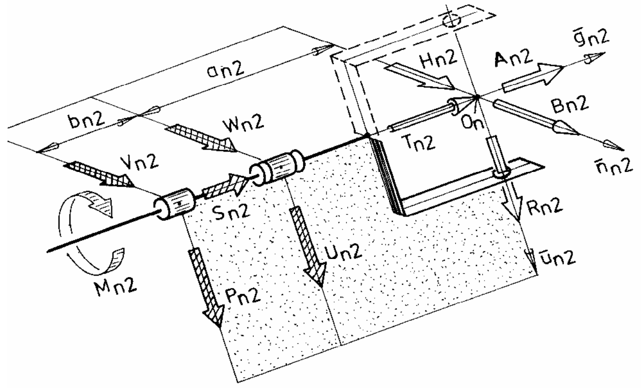
$$\begin{cases} U = -\dfrac{H_2 \cdot (a+b) + B_1 \cdot sin\,\beta - (l-a-b)\cdot (R_1 \cdot sin\,\beta - H_1 \cdot cos\,\beta)}{b}\ , \\ V = -\dfrac{B_2 - R_2 \cdot a - B_1 \cdot cos\,\beta + (l-a)\cdot (R_1 \cdot cos\,\beta + H_1 \cdot sin\,\beta)}{b}\ , \\ W = \dfrac{B_2 - R_2 \cdot (a+b) - B_1 \cdot cos\,\beta + (l-a-b)\cdot (R_1 \cdot cos\,\beta + H_1 \cdot sin\,\beta)}{b}\ . \end{cases} \tag{8}$$

For S = 0, the dependences (8) describe the *module of the correlations* between the forces of the shaft supported by a cylindrical polyjoint (Fig. 7,b); obviously, in this particular case , one of the axial forces A_1, A_2 is unknown. In the premise that the Cardan shaft (Fig. 7,b) is used in a transmission with an odd number of couplings and that in the *j* joint, the cross-fork constraints are cylindrical polyjoint, then, the radial forces of the j_1 fork become null: $H_1 = R_1 = 0$.

7. THE MODULE OF THE CORRELATIONS BETWEEN THE FORCES OF THE EXTREMAL SHAFTS

In Fig. 6 there are generally represented the forces and moments of the input shafts 11 (Fig. 6, a) and output shafts n_2 (Fig. 6, b), which are supported on the base by revolute joints. The schemes from Fig. 6 allow a simple establishment of the reactions from supports, as functions of the forks forces, which were established before. The expressions of the reactions of the two shafts, having the same form, can be written restricted, without considering the coefficients 11 and n2 (in the case of the 2-Cardan transmission 11 and n2=>22):

$$\begin{cases} T + M = J \cdot \varepsilon \ \Leftrightarrow\ T_{n2} = -M_{n2} + J_{n2} \cdot \varepsilon_{n2} \text{ şi } M_{11} = -T_{11} + J_{11} \cdot \varepsilon_{11}\ , \\ S = -A, P = -\dfrac{B - R \cdot a}{b}, V = \dfrac{H \cdot a}{b}, U = \dfrac{B - R \cdot (a+b)}{b}, W = -\dfrac{H \cdot (a+b)}{b}. \end{cases} \tag{9}$$



a.

b.

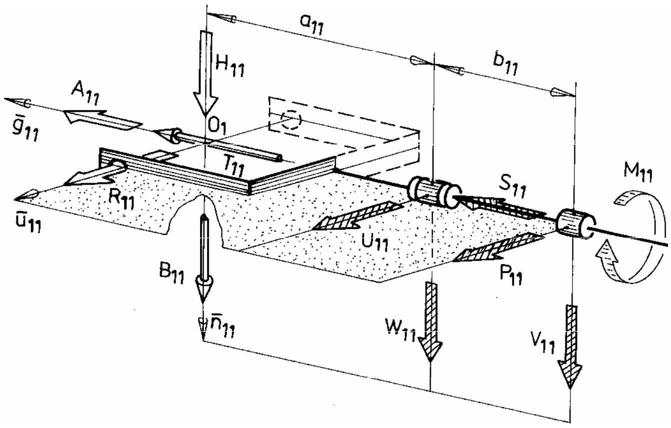


Fig. 6. The forces of extremal shafts

8. CONCLUSIONS

• A base of modular correlations, regarding the forces from the main typical modules of the Cardan transmissions, was elaborated in this paper; the less used modules will be modelled in a future paper.
• By assembling the obtained correlations it becomes possible to write an algorithm and, implicitly, to simplify the forces computational modelling. This base can be generalized for all the isostatic n-Cardan transmissions.
• Theoretically, the intermediary Cardan shafts, as the telescopic ones, have a constant length and, implicitly, the relative motion from the grooves is null; under these conditions, the friction force from the grooves has to be null, too! Under real conditions, the deviations from the theoretical model induce small relative displacements in the straight grooved connection and, implicitly, the appearance of the friction forces.
• Futher on, based on the modular correlations, it is proposed an algorithm for the dynamic computational modelling and there are presented the results of the numerical simulations for a relevant example.

References

[1] Dudiță, Fl., Diaconescu, D.V., Neagoe, M., Săulescu, R.: *On the modelling of homokinetical tricardanic transmissions*. 2nd International Conference „Research and development in mechanical industry" RaDMI 2002, Yugoslavia, ISBN 86-83803-03-1, vol II, pp. 872 – 877.
[2] Diaconescu, D., Săulescu, R., Borca, A. *About kinematic analysis and synthesis of 3-Cardan transmission*. Annals of the Oradea University. Fascicle of Management and Technological Engineering. Vol II (XII), 2003, ISSN 1583-0691, pag 24 (Abstract).
[3] Diaconescu, D.V., Săulescu. R., Borca, A. *Kinematic analysis and synthesis of cardan transmission*. Bulletin of the Transilvania University of Braşov, vol. 10 (45) - series A1, Braşov, pp.37-45, 2003, ISSN: 1223-9631.