ANNALS of the ORADEA UNIVERSITY.

Fascicle of Management and Technological Engineering, Volume VI (XVI), 2007

VIBROIMPACT REGIMES OF MOTION WITH SUPPLEMENTARY COLLISIONS STABILITY.

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Keywords: vibration, collision, stability

In the drilling equipment functioning can appear supplementary collision in a period of time that leads to the malfunction of the equipment. In this paper are analyzed periodic vibroimpact motions with a single collision during a period. So, the motions can be completely treated both from the point of view of the existence and stability of the motions having as period a multiple of the period of the periodic excitation.

The study also includes the analysis of vibroimpact motions where more than one collision during a period can appear.

1. INTRODUCTION

In this paper are treated the periodic motion regimes of vibroimpact systems for the particular situation of plastic collision. It is underlined the methodology of treatment the vibroimpact motions that is applicable to the system that have special particularities [1].

As an exemplification, the methodology will be concretized for situations in which supplementary collisions appear in each motion cycle. Showing a simple vibroimpact system leads to relieve some specific particularities and differentiation of difficulties that appears in these cases. Although the treating scheme is valid for more complex systems, forwards will be delimitated the study only for the simple case of motion with detachment of a generator of vibrations situated on a fixed rigid plane [5].

2. PERIODIC VIBROPERCUTANT REGIMES.

Periodic regimes of vibroimpact system motion are realized only for certain characteristic parameters that must be satisfied [1]. If the parameters do not satisfy the limit conditions, without sufficient restrictive values, then it will lead to the study of some complex phenomena. This is the reason for which was considered necessary the complete and detailed approach of vibroimpact system with simple periodic motions, but having supplementary collisions in each motion cycle.

Forwards is noted the moment that delimits a motion cycle by $t = t_k$ (k=1, 2…) that means that in interval $\left[t_k, t_{k+1}\right]$ is supposed that the motion is free and appear a supplementary collision.

Taking into account that in differential equation of motion appear the periodic driving force, the periodic motion regimes are possible only in the situation in which the interval between the two basal collisions is multiple of the period of driving force, i.e. $t_{k+1} - t_k = rT$ (r=1, 2, …) [3]. Other situations can appear in the cases in which more supplementary collisions intervene in each period of motion, that complete the possible periodic regimes range, but for which the effective calculus and determination are more difficult.

In this paper is considered, as an example, only the case of a single supplementary collision appearance in each motion cycle, with particularization at plastic collision case.

3. VIBROIMPACT SYSTEM

The presented study has as objective to determine the way of treatment of the multitude of situations where vibroimpact periodic regimes appear [4]. It is considered the simplified case of the generator of vibrations rest on a rigid plan that executes motions with plan detachments. Thus, in motion time the generator of vibrations lift up off plan and then, in fall collide and rebound in accordance with known laws of collision (figure 1).

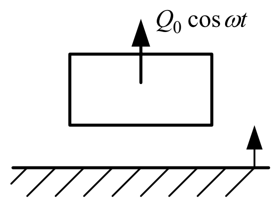


Fig. 1.

The generator of vibrations has mass *m* and driving force due to generator is by form $Q_0 \cos \omega t$. On the mass *m* act its own gravity *mg*, so the composite force will be $-mg + Q_0 \cos \omega t$ and the differential equation of motion

$$\ddot{q} = g(\delta \cos \omega t - 1) \tag{1}$$

where

$$\delta = \frac{Q_0}{mg}$$

Taking into account that in interval between considered fundamental collisions $t \in [t_k, t_{k+1}]$ appears a supplementary collision in moment $t = t' \in [t_k, t_{k+1}]$ results that each motion cycle has two phases:

- phase 1 anterior to supplementary collision,
- phase 2 posterior to supplementary collision.

In the first phase, the motion and the velocity laws are obtained by integrating the equation (1), for the plastic collision

$$\begin{gathered} q_1(t) = -\frac{g}{\omega^2}[\delta(\cos \omega t - \cos \omega t_k) + \delta\omega(t - t_k)\sin \omega t_k + \frac{1}{2}\omega^2(t - t_k)^2] \\ \dot{q}_1(t) = \frac{g}{\omega}\left[\delta(\sin \omega t - \sin \omega t_k) - \omega(t - t_k)\right] \end{gathered} \tag{2}$$

Analog, in the second phase of cycle, the motion and velocity laws are

$$\begin{gathered} q_2(t) = -\frac{g}{\omega^2}[\delta(\cos \omega t - \cos \omega t') + \delta\omega(t - t')\sin \omega t' + \frac{1}{2}\omega^2(t - t')^2] \\ \dot{q}_2(t) = \frac{g}{\omega}\left[\delta(\sin \omega t - \sin \omega t') - \omega(t - t')\right] \end{gathered} \tag{3}$$

At the establishment of laws (2) and (3) was taken into account the initial motion conditions in the two phases, so it must be determined the limit condition at the end of each phase. Thus at the end of the first phase, i.e. for $t = t'$ the limit conditions must be satisfied

$$q_1(t') = 0\,,\ \dot{q}_1(t') = \dot{q}_{10} \tag{4}$$

where $\dot{q}_{10}$ is the velocity from the beginning of supplementary collision.

Similarly, at the end of the second cycle phase, the following conditions can be written

$$q_2(t_{k+1}) = 0\,,\ \dot{q}_2(t_{k+1}) = \dot{q}_c \tag{5}$$

By introducing the limit conditions (4) in the motion and velocity laws (2), it results

$$\begin{gathered} q_1(t') = -\frac{g}{\omega^2}[\delta\left(\cos\omega t' - \cos\omega t_k\right) + \delta\omega(t'-t_k)\sin\omega t_k + \frac{1}{2}\omega^2(t'-t_k)^2] = 0 \\ \dot{q}_1(t') = \frac{g}{\omega}\left[\delta\left(\sin\omega t' - \sin\omega t_k\right) - \omega(t'-t_k)\right] = \dot{q}_{10} \end{gathered} \tag{6}$$

In the same manner are introduced the motion laws (3) in limit conditions (5) and results

$$\begin{gathered} q_2(t_{k+1}) = -\frac{g}{\omega^2}[\delta\left(\cos\omega t_{k+1} - \cos\omega t'\right) + \delta\omega(t_{k+1}-t')\sin\omega t' + \frac{1}{2}\omega^2(t_{k+1}-t')^2] = 0 \\ \dot{q}_2(t_{k+1}) = \frac{g}{\omega}\left[\delta\left(\sin\omega t_{k+1} - \sin\omega t'\right) - \omega(t_{k+1}-t')\right] = \dot{q}_c \end{gathered} \tag{7}$$

4. THE STABILITY OF VIBROIMPACT MOTIONS

Starting from the motion lows (2)-(3), at the initial moment it can be written

$$\begin{gathered} \delta(\cos\omega t' - \cos\omega t_k) + \delta\omega(t'-t_k)\sin\omega t_k + \frac{1}{2}\omega^2(t'-t_k)^2 = 0 \\ \delta(\cos\omega t_{k+1} - \cos\omega t') + \delta\omega(t_{k+1}-t')\sin\omega t' + \frac{1}{2}\omega^2(t_{k+1}-t')^2 = 0 \end{gathered} \tag{8}$$

Considering small perturbations $t'+\Delta t'$, $t_k+\Delta t_k$ and $t_{k+1}+\Delta t_{k+1}$, the above system becomes

$$\delta\left[\cos\omega(t'+\Delta t') - \cos\omega(t_k+\Delta t_k)\right] + \delta\omega(t'+\Delta t'-t_k-\Delta t_k)\sin\omega(t_k+\Delta t_k) +$$
$$+\frac{1}{2}\omega^2(t'+\Delta t'-t_k-\Delta t_k)^2 = 0$$

$$\delta\left[\cos\omega(t_{k+1}+\Delta t_{k+1}) - \cos\omega(t'+\Delta t')\right] + \delta\omega(t_{k+1}+\Delta t_{k+1}-t'-\Delta t')\sin\omega(t'+\Delta t') +$$
$$+\frac{1}{2}\omega^2(t_{k+1}+\Delta t_{k+1}-t'-\Delta t')^2 = 0$$

That can be also written

$$\delta(\cos\omega t'\cos\omega\Delta t' - \sin\omega t'\sin\omega\Delta t' - \cos\omega t_k\cos\omega\Delta t_k + \sin\omega t_k\sin\omega\Delta t_k) +$$
$$+\delta\omega(t'+\Delta t'-t_k-\Delta t_k)(\sin\omega t_k\cos\omega\Delta t_k + \cos\omega t_k\sin\omega\Delta t_k) + \frac{1}{2}\omega^2(t'+\Delta t'-t_k-\Delta t_k)^2 = 0$$

$$\delta(\cos\omega t_{k+1}\cos\omega\Delta t_{k+1} - \sin\omega t_{k+1}\sin\omega\Delta t_{k+1} - \cos\omega t'\cos\omega\Delta t' + \sin\omega t'\sin\omega\Delta t') +$$
$$+\delta\omega(t_{k+1}+\Delta t_{k+1}-t'-\Delta t')(\sin\omega t'\cos\omega\Delta t' + \cos\omega t'\sin\omega\Delta t') + \frac{1}{2}\omega^2(t_{k+1}+\Delta t_{k+1}-t'-\Delta t')^2 = 0$$

Due to the fact that the perturbations are small, converge to zero, it can be written

$$\cos\omega\Delta t' = \cos\omega\Delta t_k = 1,$$

$$\sin\omega\Delta t' = \omega\Delta t' ,$$
$$\sin\omega\Delta t_k = \omega\Delta t_k ,$$
$$\sin\omega\Delta t_{k+1} = \omega\Delta t_{k+1}$$

It results

$$\delta(\cos\omega t' - \omega\Delta t'\sin\omega t' - \cos\omega t_k + \omega\Delta t_k \sin\omega t_k) + \delta\omega(t' + \Delta t' - t_k - \Delta t_k)(\sin\omega t_k + \omega\Delta t_k \cos\omega t_k) +$$
$$+\frac{1}{2}\omega^2(t' + \Delta t' - t_k - \Delta t_k)^2 = 0$$

$$\delta(\cos\omega t_{k+1} - \omega\Delta t_{k+1}\sin\omega t_{k+1} - \cos\omega t' + \omega\Delta t'\sin\omega t') + \delta\omega(t_{k+1} + \Delta t_{k+1} - t' - \Delta t')(\sin\omega t' + \omega\Delta t'\cos\omega t') +$$
$$+\frac{1}{2}\omega^2(t_{k+1} + \Delta t_{k+1} - t' - \Delta t')^2 = 0$$

From these equations are subtracted the initial equations, so it results

$$\delta(-\omega\Delta t'\sin\omega t' + \omega\Delta t_k \sin\omega t_k) + \delta\omega(\Delta t' - \Delta t_k)\sin\omega t_k + \delta\omega(t' - t_k)\omega\Delta t_k \cos\omega t_k +$$
$$+\omega^2(t' - t_k)(\Delta t' - \Delta t_k) = 0$$

$$\tag{9}$$

$$\delta(-\omega\Delta t_{k+1}\sin\omega t_{k+1} + \omega\Delta t'\sin\omega t') + \delta\omega(\Delta t_{k+1} - \Delta t')\sin\omega t' + \delta\omega(t_{k+1} - t')\omega\Delta t'\cos\omega t' +$$
$$+\omega^2(t_{k+1} - t')(\Delta t_{k+1} - \Delta t') = 0$$

In order to simplify the calculus, the following notation were introduced

$$\omega(t' - t_k) = u \tag{10}$$
$$\omega(t_{k+1} - t') = \omega(t_{k+1} - t_k) - \omega(t' - t_k) = 2\pi r - u$$

And equation (9) becomes

$$\delta(-\omega\Delta t'\sin\omega t' + \omega\Delta t_k \sin\omega t_k) + \delta\omega(\Delta t' - \Delta t_k)\sin\omega t_k + \delta u\omega\Delta t_k \cos\omega t_k + \omega u(\Delta t' - \Delta t_k) = 0$$

$$\delta(-\omega\Delta t_{k+1}\sin\omega t_k + \omega\Delta t'\sin\omega t') + \delta\omega(\Delta t_{k+1} - \Delta t')\sin\omega t' + \delta(2\pi r - u)\omega\Delta t'\cos\omega t' +$$
$$+\omega(2\pi r - u)(\Delta t_{k+1} - \Delta t') = 0$$

And it results

$$\frac{\Delta t_{k+1}}{\Delta t_k} = \frac{u(\delta\cos\omega t_k - 1)\left[(2\pi r - u) - \delta(2\pi r - u)(\cos u\cos\omega t_k - \sin u\sin\omega t_k)\right]}{(\delta\sin u\cos\omega t_k + \delta(\cos u - 1)\sin\omega t_k - u)} \cdot$$
$$\cdot\frac{1}{\left[\delta(\cos u - 1)\sin\omega t_k + \delta\sin u\cos\omega t_k + (2\pi r - u)\right]} \tag{11}$$

5. CONCLUSIONS

As it was shown in a previous paper [2] it results that equation (11) does not depend on parameter δ.

$$\sin\omega t_k = \frac{BF - DE}{\delta(AD - BC)}$$
$$\cos\omega t_k = \frac{CE - AF}{\delta(AD - BC)}$$

where

$$A = u - \sin u ,$$

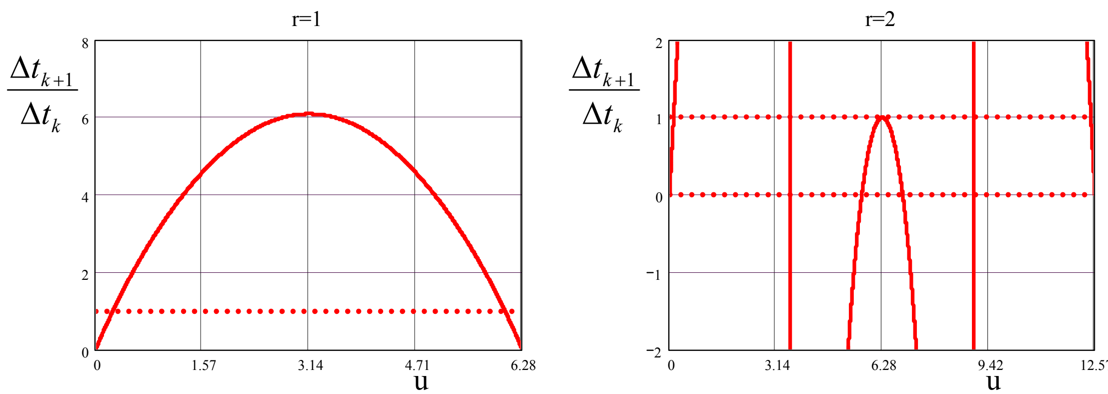
$$B = -1 + \cos u\,,$$

$$C = u + (2\pi r - u)\cos u\,,$$

$$D = (2\pi r - u)\sin u\,,$$

$$E = \frac{1}{2}u^2\,,$$

$$F = u^2 - 2\pi r u + 2\pi^2 r^2$$



It can be observed that with the increasing of the period (r=2) the stability domain $\frac{\Delta t_{k+1}}{\Delta t_k} < 1$ is smaller.

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