

STANDARD PRACTICE FOR STATISTICAL ANALYSIS OF LINEARIZED STRAIN-LIFE (ϵ -N) FATIGUE DATA, FOR 16Mo5.3b STEEL SUBMITTED TO THERMOMECHANICAL FATIGUE

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Using a modern process of experimental datas statistical analysis is tracing the material thermomechanical fatigue curve. This paper presents thermomechanical fatigue tests of 16Mo5.3b steel test specimens, in temperature range $60^{\circ}\text{C} \div 540^{\circ}\text{C}$, with total strain control in which cycle of temperature variation, on ring surface test specimens. Thermal condition of test is with soaking period at maximum temperature 540°C .

1. STANDARD PRACTICE FOR STATISTICAL ANALYSIS OF LINEAR OR LINEARIZED STRESS-LIFE (S-N) AND STRAIN-LIFE (ϵ -N) FATIGUE DATA

Material scientists and engineers are making increased use of statistical analyses in interpreting stress-life (S-N) and strain-life (ϵ -N) fatigue data. It is well known that the shape of (S-N) and (ϵ -N) curves can depend markedly on the material and test conditions. This practice is restricted to linear or linearized (S-N) and (ϵ -N) relationships, for example:

$$\log N = A + B \log \epsilon \quad (1)$$

$$\log N = A + B \log S \quad (2)$$

in which S and ϵ may refer to the amplitude or the range of the constant amplitude cyclic stress or strain, given a specific value of the mean stress or strain. In fact, the range of total strain was actually controlled during testing.

The maximum likelihood estimators of A and B are as follows [5]:

$$\hat{A} = \bar{Y} - \hat{B} \cdot \bar{X} \quad (3)$$

$$\hat{B} = \frac{\sum_{i=1}^k (X_i - \bar{X}) \cdot (Y_i - \bar{Y})}{\sum_{i=1}^k (X_i - \bar{X})^2} \quad (4)$$

where: the symbol „caret”(^) denotes estimate (estimator)

the symbol „overbar”(¯) denotes average, for example:

$$\bar{Y} = \sum_{i=1}^k \frac{Y_i}{k} ; \quad Y_i = \log N_i \quad (5)$$

$$\bar{X} = \sum_{i=1}^k \frac{X_i}{k} ; \quad X_i = \log \epsilon_i \quad (6)$$

k is the total number of the test specimens.

The recommended expression for estimating the variance of the normal distribution for log N is:

$$\hat{\sigma}^2 = \frac{\sum_{i=1}^k (Y_i - \hat{Y}_i)^2}{k-2} \quad (7)$$

in which: $\hat{Y}_i = \hat{A} + \hat{B} \cdot X_i$

Confidence intervals for parameters A and B can be established using the „t” (student) distribution:

$$\hat{A} \pm t_p \cdot \hat{\sigma}_{\hat{A}} \quad (8)$$

or:

$$\hat{A} \pm t_p \cdot \hat{\sigma} \cdot \left[\frac{1}{k} + \frac{\bar{X}^2}{\sum_{i=1}^k (X_i - \bar{X})^2} \right]^{1/2} \quad (9)$$

and for B:

$$\hat{B} \pm t_p \cdot \hat{\sigma}_{\hat{B}} \quad (10)$$

or:

$$\hat{B} \pm t_p \cdot \hat{\sigma} \cdot \left[\sum_{i=1}^k (X_i - \bar{X})^2 \right]^{-1/2} \quad (11)$$

in which t_p value is read from Table 1, for the desired value of P, the confidence level associated with the confidence interval [5].

Values of t_p

Table 1.

n = k-2 statistical degrees of freedom	Value of t_p P % = probability that the random variable t lies in the interval from $-t_p$ to $+t_p$	
	P = 90%	P = 95%
4	2,1318	2,7464
5	2,0150	2,5706
6	1,9432	2,4469
7	1,8946	2,3646
8	1,8595	2,3060
9	1,8331	2,2622
10	1,8125	2,2281
20	1,7247	2,0860

The confidence band for the entire median S-N or ε -N curve may be computed using the following equation:

$$\hat{A} + \hat{B} \cdot X \pm \sqrt{2 \cdot F_p} \cdot \hat{\sigma} \cdot \left[\frac{1}{k} + \frac{(X - \bar{X})^2}{\sum_{i=1}^k (X_i - \bar{X})^2} \right]^{1/2} \quad (12)$$

in which F_p is given in Table 2.

Values of F_p

Table 2

$n_2 = k_2 - 2$ degrees of freedom	$n_1 = k_1 - 2$			
	1	2	3	4
1	161,45	199,50	215,71	224,58
	4052,2	4999,5	5403,3	5624,6
2	18,513	19,000	19,164	19,247
	98,503	99,000	99,166	99,249
3	10,128	9,5521	9,2766	9,1172
	34,166	30,817	29,457	28,710
4	7,7086	6,9443	6,5914	6,3883
	21,198	18,000	16,694	15,977
5	6,6079	5,7861	5,4095	5,1922
	16,258	13,274	12,060	11,392
6	5,9874	5,1433	4,7571	4,5337
	13,745	10,925	9,7795	9,1483
7	5,5914	4,7374	4,3468	4,1203
	12,246	9,5466	8,4513	7,8467
8	5,3177	4,4590	4,0662	3,8378
	11,259	8,6491	7,5910	7,0060
9	5,1174	4,2565	3,8626	3,6331
	10,561	8,0215	6,9919	6,4221
10	4,9646	4,1028	3,7083	3,4780
	10,044	7,5594	6,5523	5,9943
11	4,8443	3,9823	3,5874	3,3567
	9,6460	7,2057	6,2167	5,6683
12	4,7472	3,8853	3,4903	3,2592
	9,3302	6,9266	5,9526	5,4119

In each row, the top figures are values of F_p corresponding to $P = 95\%$, the bottom figures correspond to $P = 99\%$. Thus, the top figures pertain 5% significance level, whereas the bottom figures pertain 1% significance level. The bottom figures are not recommended for use in equation 12.

2. NUMERICAL EXAMPLE. DATA PROCESSING

The thermomechanical fatigue tests from 16 Mo 5.3b are conducted in temperature range $60^\circ \leftrightarrow 540^\circ\text{C}$, with total strain control ($\Delta\varepsilon_{\text{total}} = \text{const}$), in wich cycle of temperature variation [1],[2],[3]. Thermal conditions of tests are with 3600 second soaking period at maximum temperature of 540°C . The test specimen, pending testing, is shown on fig.1.



Fig.1. Test specimen from 16Mo5.3b, pending testing

On nest set of six test specimens tested to thermomechanical fatigue are shown on fig. 2.



Fig. 2. Test specimens tested to thermomechanical fatigue

The primarys test datas obtained to thermomechanical fatigue tests of 16Mo5.3b, saw-toothed conditions inter $60^{\circ} \Leftrightarrow 540^{\circ}\text{C}$, are shown on Table 3. Are presented pair of values from range (amplitude) of total strain dependent on the fatigue life N , number of cycles from crack.

The primary test datas from 16Mo5.3b, inter $60^{\circ} \Leftrightarrow 540^{\circ}\text{C}$, with 3600 second soaking period at maximum temperature of 540°C

Table 3

Number of test specimen	1	2	3	4	5	6	7	8
$\Delta\epsilon_t$ [-]	0,0042	0,0042	0,006	0,006	0,0081	0,0081	0,0105	0,0105
$\Delta\epsilon_t$ [%]	0,42	0,42	0,6	0,6	0,81	0,81	1,05	1,05
N [cycles]	664	591	158	165	150	134	60	51

In Table 4 are presented the results of statistical analysis, in accordance with standard practice [5].

The results of statistical analysis:

Table 4

Nr. crt	$\Delta\epsilon$ [%]	N_f	$\Delta\epsilon$ [-]	$X_i = \lg \Delta\epsilon_t$	$Y_i = \lg N_{fi}$	$(X_i - \bar{X})$	$(Y_i - \bar{Y})$
1	0,42	664	0,0042	-2,3768	2,8222	-0,2018	+0,5973
2	0,42	591	0,0042	-2,3768	2,7716	-0,2018	+0,5467
3	0,6	158	0,006	-2,2218	2,1987	-0,0571	-0,0262
4	0,6	165	0,006	-2,2218	2,2175	-0,0571	-0,0074

5	0,81	150	0,0081	-2,0915	2,1761	+0,0732	-0,0488
6	0,81	134	0,0081	-2,0915	2,1271	+0,0732	-0,0978
7	1,05	60	0,0105	-1,9788	1,7782	-0,1859	-0,4467
8	1,05	51	0,0105	-1,9788	1,7075	+0,1859	-0,5174

(continuation table)

Nr. crt	$\Delta\epsilon$ [%]	$(X_i - \bar{X}) \cdot (Y_i - \bar{Y})$	$(X_i - \bar{X})^2$	$\hat{Y} = \hat{A} + \hat{B} \cdot X_i$	$(Y_i - \hat{Y})^2$
1	0,42	-0,1205	0,0407	3,0249	0,0097
2	0,42	-0,1103	0,0407	3,0249	0,0023
3	0,6	+0,0015	0,0033	3,2697	0,0279
4	0,6	+0,0042	0,0033	3,2697	0,0221
5	0,81	-0,0036	0,0054	2,9427	0,0175
6	0,81	-0,0072	0,0054	2,9427	0,0069
7	1,05	-0,0830	0,0346	2,6643	0,0002
8	1,05	-0,0962	0,0346	2,6643	0,0034

The total number of test specimens tested: $n = 8$; degrees of freedom $k = n - 2 = 6$;

Sumes calculations:

$$\sum X_i = -2,1647$$

$$\sum Y_i = +2,2249;$$

$$\sum (X_i - \bar{X}) = +0,0004;$$

$$\sum (Y_i - \bar{Y}) = -0,0003;$$

$$\sum (X_i - \bar{X}) \cdot (Y_i - \bar{Y}) = -0,4151;$$

$$\sum (X_i - \bar{X})^2 = +0,1680;$$

$$\sum (Y_i - \hat{Y})^2 = +0,0900$$

Ancillary calculations:

$$\hat{B} = \frac{\sum (X_i - \bar{X}) \cdot \sum (Y_i - \bar{Y})}{\sum (X_i - \bar{X})^2} = \frac{-0,4151}{0,168} = -2,4708$$

$$\hat{A} = \bar{Y} - \hat{B} \cdot \bar{X} = 2,2249 - (-2,4708) \cdot (-2,1647) = -3,1236$$

$$\hat{\sigma}^2 = \frac{\sum (Y_i - \hat{Y})^2}{k - 2} = \frac{0,09}{6} = 0,015; \rightarrow \hat{\sigma} = 0,1225;$$

$F_p = 5,1433$ for $n_1 = 2$ and $n_2 = 6$ for probability 5% and 95%.

Average value, value at minimum limit and superior limit:

For $\Delta\epsilon = 1,05$ %:

$$\hat{Y} = \bar{Y} - \hat{B} \cdot \bar{X} + \sqrt{2F_p} \cdot \hat{\sigma} \cdot \sqrt{\frac{1}{k} + \frac{(X - \bar{X})^2}{\sum (X_i - \bar{X})^2}} = -3,1236 + [(-2,4708) \cdot (-1,9586)] \pm$$

$$\pm \sqrt{2 \cdot 5,143} \cdot 0,1225 \cdot \sqrt{\frac{1}{6} + \frac{(-1,9586 + 2,1647)^2}{0,168}} = 1,7157 \pm 0,2546$$

$$\rightarrow \hat{Y}_{sup} = 1,9703 \rightarrow N_{fsup} = 93$$

$$\rightarrow \hat{Y}_{inf} = 1,4611 \rightarrow N_{finf} = 29$$

For $\Delta\epsilon = 0,81 \%$

$$\hat{Y} = \bar{Y} - \hat{B} \cdot \bar{X} + \sqrt{2F_p} \cdot \hat{\sigma} \cdot \sqrt{\frac{1}{k} + \frac{(X - \bar{X})^2}{\sum (X_i - \bar{X})^2}} = -3,1236 + [(-2,4708) \cdot (-2,0969)] \pm$$

$$\pm \sqrt{2 \cdot 5,143} \cdot 0,1225 \cdot \sqrt{\frac{1}{6} + \frac{(-2,0969 + 2,1647)^2}{0,168}} = 2,0574 \pm 0,1732$$

$$\rightarrow \hat{Y}_{sup} = 2,231 \rightarrow N_{fsup} = 170$$

$$\rightarrow \hat{Y}_{inf} = 1,8850 \rightarrow N_{finf} = 77$$

For $\Delta\epsilon = 0,6 \%$:

$$\hat{Y} = \bar{Y} - \hat{B} \cdot \bar{X} + \sqrt{2F_p} \cdot \hat{\sigma} \cdot \sqrt{\frac{1}{k} + \frac{(X - \bar{X})^2}{\sum (X_i - \bar{X})^2}} = -3,1236 + [(-2,4708) \cdot (-2,2218)] \pm$$

$$\pm \sqrt{2 \cdot 5,143} \cdot 0,1225 \cdot \sqrt{\frac{1}{6} + \frac{(-2,2218 + 2,1647)^2}{0,168}} = 2,3660 \pm 0,1696$$

$$\rightarrow \hat{Y}_{sup} = 2,5356 \rightarrow N_{fsup} = 343$$

$$\rightarrow \hat{Y}_{inf} = 2,1964 \rightarrow N_{finf} = 157$$

For $\Delta\epsilon = 0,42 \%$:

$$\hat{Y} = \bar{Y} - \hat{B} \cdot \bar{X} + \sqrt{2F_p} \cdot \hat{\sigma} \cdot \sqrt{\frac{1}{k} + \frac{(X - \bar{X})^2}{\sum (X_i - \bar{X})^2}} = -3,1236 + [(-2,4708) \cdot (-2,3979)] \pm$$

$$\pm \sqrt{2 \cdot 5,143} \cdot 0,1225 \cdot \sqrt{\frac{1}{6} + \frac{(-2,3979 + 2,1647)^2}{0,168}} = 2,8011 \pm 0,2752$$

$$\rightarrow \hat{Y}_{sup} = 3,0763 \rightarrow N_{fsup} = 1.192$$

$$\rightarrow \hat{Y}_{inf} = 2,5259 \rightarrow N_{finf} = 336$$

The medium line regression and bands of dispersion are calculated in Table 5.

The medium line regression and bands of dispersion

Table 5

Nr. crt	$\Delta\epsilon$ [%]	$\Delta\epsilon$ [-]	X	$\hat{Y} = \hat{A} + \hat{B} \cdot X$	N_{imed} [cycles]	$\hat{Y}_{sup}$	N_{fsup} [cycles]	$\hat{Y}_{inf}$	N_{finf} [cycles]
1	1,05	0,0105	-1,9586	1,7157	52	1,9703	93	1,4611	29
2	0,81	0,0081	-2,0969	2,0574	114	2,231	170	1,8850	77
3	0,6	0,006	-2,2218	2,3660	232	2,5356	343	2,1964	157
4	0,42	0,0042	-2,3979	2,8011	633	3,0763	1.192	2,5259	336

The equation of line regression, Manson-Coffin Equation,[1],[2],[3],[4],[5], for probability $P = 50\%$ (e-med) is:

$$y = 4.1487 \cdot x^{-0.3126} \quad \text{or} \quad \Delta \epsilon_{tot} = 4.1487 \cdot N_f^{-0.3126}$$

where: $\Delta \epsilon_{tot}$ is the range (amplitude) of total strain [%];
 N_f - number of cycles from crack.

In figure 3 is the thermomechanical fatigue curve, strain-life (ϵ -N), Manson-Coffin Equation, 16Mo5.3b steel. The thermomechanical fatigue tests from 16 Mo 5.3b are conducted in temperature range $60^\circ \leftrightarrow 540^\circ\text{C}$, with total strain control ($\Delta \epsilon_{total} = \text{const}$), in which cycle of temperature variation [1],[2],[3]. Thermal conditions of tests are with 3600 second soaking period at maximum temperature of 540°C .

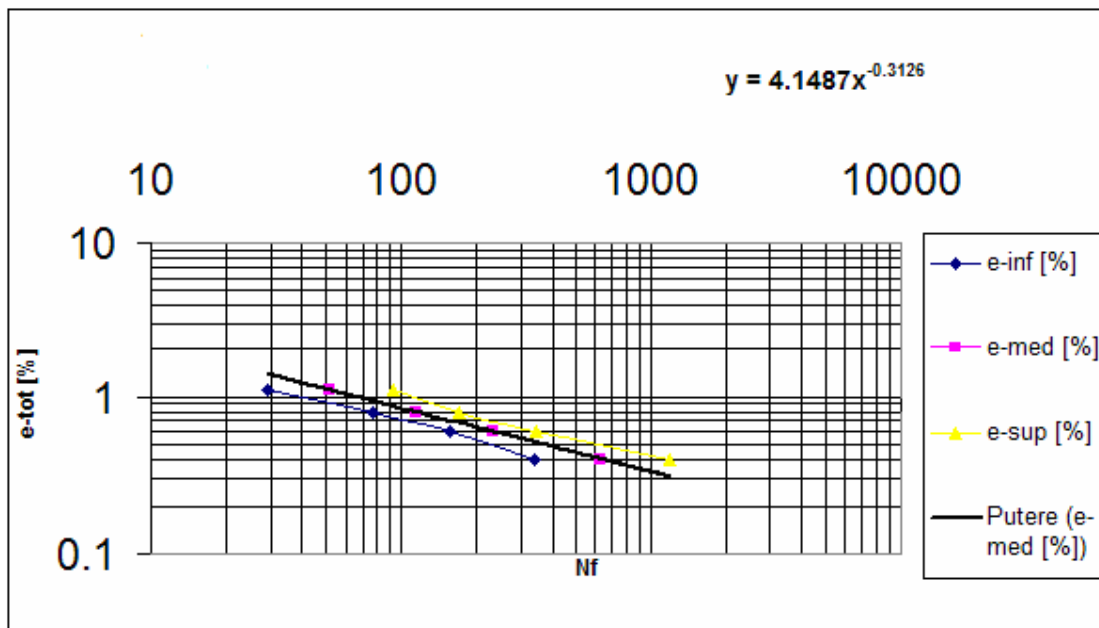


Fig.3. Manson-Coffin Equation from 16Mo5.3b steel, the thermomechanical fatigue tests are conducted in temperature range $60^\circ \leftrightarrow 540^\circ\text{C}$, with 3600 second soaking period at maximum temperature of 540°C .

This practice pertains only to (ϵ -N) relationships what may be reasonably approximated by a straight line for a specific interval of stress or strain. It presents elementary procedures that presently reflect good practice in modeling and analysis.

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