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MATHEMATICAL MODELLING THE FORCE-DISPLACEMENT DEPENDENCE FOR EXPANDED METAL THREADS

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Abstract. The reduced Bézier curves provide us with an analytical representation of the force-displacement dependence in case of a plane plaque made out of standard expanded metal threads.

1. Introduction

The aim of this paper is to elaborate a mathematical model, to study the force-displacement dependence when submitting to an external force a plane plaque of expanded metal made out of expanded metal threads (fig. 1).

Fig. 1. ***Expanded metal thread***

Expanded Metal comes out of sheets of solid metal that are slit and stretched with each stroke of the upper die, forming a raised diamond pattern. This pattern is called regular or standard. The pattern varies by the gauge and type of material and the size of the diamond. The sheets can also be flattened by rollers for an optional style by passing them through a cold, roll reducing mill. In this paper we deal with expanded metal plane standard structures, as in figure 2, which are a succession of cells bordered by expanded metal threads.

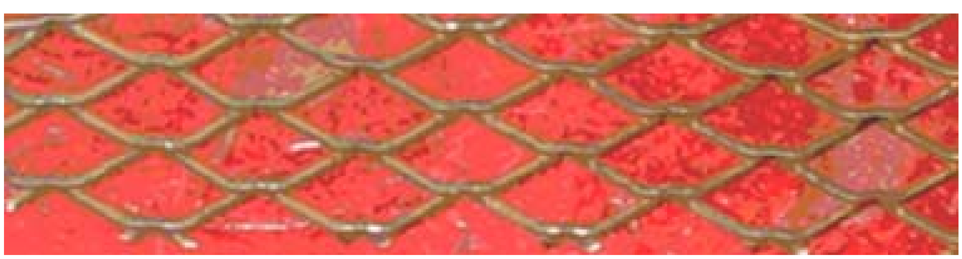

Fig. 2. ***Plane structure of expanded metal made aut of five threads***

We use the example illustrated in figure 2, which is obtained by an expanding process using triangular knives.The other types of expanded metal are easy to study in an analogous manner.

The model bases on reduced Bézier curves in order to get a simple overall description of the variation of those parameters. The problem of the degree reduction of a Bézier curve is solved in paper [1], when continuity conditions are states at the extremities, giving an algorithm, which we are going to use for our purposes. This algorithm was formulated in matrix form in paper [5]. Its result is an approximate Bézier curve that preserves the behaviour of the approximated function, in terms of monotony and convexity / concavity, as discussed in [2].

Let us consider the set of control points $\{p_i$: i=0,1,...,n$\}$. A degree n Bézier curve is defined by

$$p(t) = \sum_{i=0}^{n} p_i B_i^n(t), \quad t \in [0,1], \tag{1.1}$$

where

$$B_i^n(t) = \binom{n}{i} t^i (1-t)^{n-i} \tag{1.2}$$

is the Bernstein polynomial of degree n. The problem of the degree reduction of a Bézier curve is solved in paper [1], when continuity conditions are stated at the extremities, giving an algorithm, which we are going to use for our purposes. It consists in finding control points $\{q_i$: i=0,1,...,m$\}$, which define the approximate Bézier curve

$$q(t) = \sum_{i=0}^{m} q_i B_i^m(t), \quad t \in [0,1], \tag{1.3}$$

of degree m(m<n) such that a suitable distance function d(p,q) is minimized. Here, the (L_2) norm is used. We define the optimal multi-degree reduction with constraints of endpoints continuity as in [1].

These curves have the advantage of preserving the main shape properties of the functions giving this dependence. Therefore, the information on the monotony, convexity, concavity of those functions is accurate, if they are approximated by Bézier curves. On the other hand, these curves are polynomials, which are easier to use for further applications than the splines, for example. On the other hand, they provide a convergent model, correcting the defect of the Lagrange operators.

2. Data collecting

The data involved in this approach are collected within the Laboratory of Materials Resistance of the Faculty of Engineering of "Aurel Vlaicu" University of Arad in 2005 and are included in [3].

Table 1 contains the values of forces, when the expanded metal plaques made out of 1, 2, 3, 5, 7, respectively 9 threads are submitted to vertical displacements of 0.5 mm step. The plaques are fixed at the extremities of the threads.

Displ. [mm]	Force [N] for the considered number of threads					
	1	2	3	5	7	9
0,5	18.387	36.775	61.292	85.808	110.32	134.84
1,0	24.517	49.033	85.808	110.32	140.97	171.62
1,5	27.581	67.421	98.067	128.71	165.49	208.39
2,0	30.646	79.679	110.32	147.1	196.13	239.04
2,5	30.646	98.067	134.84	196.13	232.91	269.68
3,0	33.71	116.45	159.36	232.91	269.68	318.72
3,5	36.775	134.84	183.87	257.42	294.2	349.36
4,0	42.904	171.62	208.39	294.2	343.23	380.01
4,5	61.292	196.13	232.91	318.72	367.75	422.91
5,0	76.614	226.78	269.68	349.36	392.27	459.69
5,5	85.808	251.3	294.2	380.01	416.78	484.2
6,0	91.937	281.94	312.59	416.78	447.43	514.85
6,5	98.067	306.46	330.97	453.56	478.07	557.75
7,0	104.2	330.97	380.01	484.2	514.85	600.66
7,5	110.32	361.62	416.78	514.85	551.62	661.95
8,0	116.45	392.27	447.43	545.49	612.92	723.24
8,5	128.71	422.91	478.07	576.14	661.95	809.05
9,0	140.97	441.3	514.85	600.66	710.98	845.82
9,5	153.23	459.69	551.62	661.95	772.27	907.12
10,0	171.62	478.07	576.14	710.98	809.05	968.41
10,5	190	490.33	612.92	772.27	870.34	1017.4
11,0	214.52	502.59	637.43	833.57	931.63	1042
11,5	245.17	514.85	698.72	894.86	992.92	1078.7
12,0	281.94	539.37	735.5	931.63	1054.2	1140
12,5	306.46	563.88	809.05	968.41	1103.2	1176.8
13,0	330.97	588.4	858.08	1029.7	1140	1213.6
13,5	361.62	600.66	894.86	1066.5	1189.1	1287.1
14,0	392.27	612.92	943.89	1127.8	1225.8	1348.4
14,5	416.78	637.43	980.67	1164.5	1287.1	1440.4
15,0	429.04	649.69	1029.7	1189.1	1440.4	1532.3
15,5	441.3	661.95	1054.2	1225.8	1501.6	1593.6
16,0	453.56	686.47	1078.7	1287.1	1562.9	1685.5
16,5	459.69	710.98	1127.8	1379.1	1654.9	1777.5
17,0	465.82	723.24	1176.8	1471	1716.2	1869.4
17,5	465.82	747.76	1213.6	1562.9	1777.5	1992
18,0	471.95	784.53	1287.1	1716.2	1869.4	2083.9

Tab. 1. ***Values of forces corresponding to various values of displacements***

3. Mathematical modelling by reduced Bézier curves

The algorithm of Chen and Wang, is used in this section as it was formulated in matrix form in paper [5]. The endpoints continuity conditions are of the following type:

$$\frac{d^i q(0)}{dt^i} = \frac{d^i p(0)}{dt^i}, \quad i = 0,1,...,r;$$
$$\frac{d^j q(1)}{dt^j} = \frac{d^j p(1)}{dt^j}, \quad j = 0,1,...,s.$$

In the particular case of applying a force on a single expanded metal thread (EMT), the control points p = (p_x, p_y) have the following significance: the first coordinate means the displacement produced by the force placed on the position of the second coordinate. The control points are:

p_0 = (0, 0), p_1 = (2, 30.646), p_2 = (4, 42.904), p_3 = (6, 91.937),
p_4 = (8, 116.45), p_5 = (10, 140.97), p_6 = (12, 281.94),
p_7 = (14, 392.27), p_8 = (16, 453.56), p_9 = (18, 471.95).

These points generate a Bézier curve of degree 9 by formula (1.1), together with (1.2). We take m = 3 and r = s = 0, which means the coincidence of the values on the first and on the last point and the degree reduction from 9 to 3. Step 1 of the Chen and Wang algorithm gives us

q_0 = (0, 0) = p_0 and q_3 = (18, 471.95) = p_9,

as required. The transition matrices are:

$$C = \begin{pmatrix} -0.06 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.012 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.048 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.129 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.238 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.417 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.66 \end{pmatrix},$$

$$I_{2\times 8} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix},$$

$$D = \begin{pmatrix} 9 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 5.143 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3.6 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3.6 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 5.143 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 9 \end{pmatrix},$$

$$E_{2\times 2} = \begin{pmatrix} 0.5 & 0.5 \\ -0.167 & -0.167 \end{pmatrix},$$

$$D^* = \begin{pmatrix} 0 & 0 \\ 0.33 & 0 \\ 0 & 0.33 \\ 0 & 0 \end{pmatrix},$$

The new knots are:

$q_0 = p_0$, $q_1 = (-0.139, -4.441)$, $q_2 = (-0.177, -4.925)$, $q_3 = p_9$.

We wrote the Bézier curve on the new points, denoting it by $Q_0(t)$. The reduced Bézier curve is the dashed blue line in figure 3, while the continuous red line depicts the non-reduced curve.

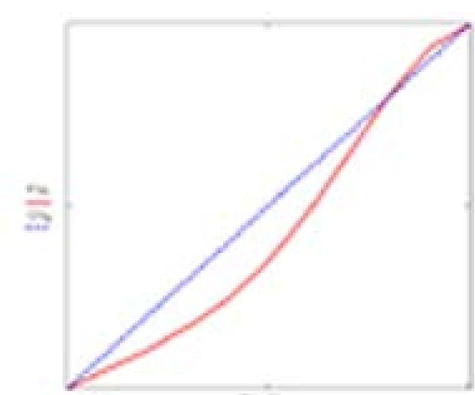

Fig. 3. The displacement-force dependence for one EMT

The case of two threads is treated in the same manner. The initial control points are:

$p_0 = (0, 0)$, $p_1 = (2, 76.679)$, $p_2 = (4, 171.62)$, $p_3 = (6, 281.94)$, $p_4 = (8, 392.27)$, $p_5 = (10, 478.07)$, $p_6 = (12, 539.37)$,

$p_7 = (14, 612.92)$, $p_8 = (16, 686.47)$, $p_9 = (18, 784.53)$.

After performing the matrix transformations according to [1] and [5], one gets the following final control points, reducing the Bézier curve from degree 9 to degree 3:

$q_0 = p_0$, $q_1 = (-0.139, -5.364)$, $q_2 = (-0.177, -6.689)$, $q_3 = p_9$

Figure 4 represents the displacement – force dependence in the case of a plane plaque made out of two EMT, using the same colour convention as in figure 3.

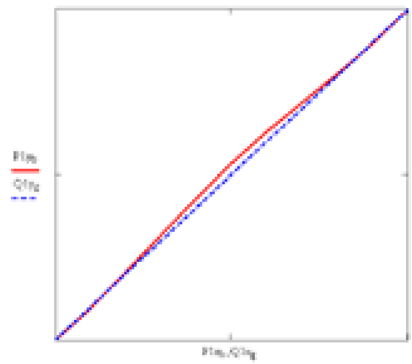

Fig. 4. The case of 2 EMT

If the expanded metal plane plaque is made out of five EMT, then the initial control points:

$p_0 = (0, 0)$, $p_1 = (2, 147.1)$, $p_2 = (4, 29462)$, $p_3 = (6, 416.78)$,
$p_4 = (8, 545.49)$, $p_5 = (10, 710.98)$, $p_6 = (12, 931.63)$,
$p_7 = (14, 1127.8)$, $p_8 = (16, 1287.1)$, $p_9 = (18, 1716.2)$

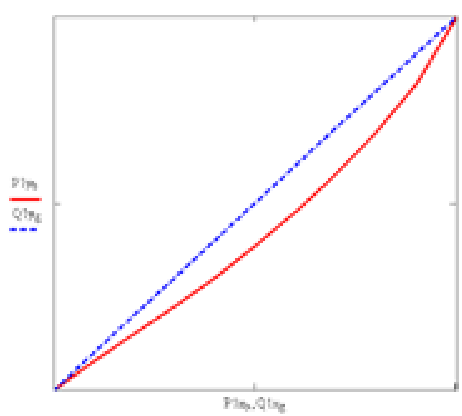

Fig. 5. The case of 5 EMT

become the final control points generating degree 3 the reduced curve:

$q_0 = p_{0,}$ $\quad q_1 = (-0.139, -10.01),$ $\quad q_2 = (-0.177, -14.411),$ $\quad q_3 = p_9$

The corresponding graph comparison is depicted in figure 5.

If a plane surface made out of nine EMT is taken into account, then the initial control points:

$p_0 = (0, 0),$ $\quad p_1 = (2, 239.04),$ $\quad p_2 = (4, 380.01),$ $\quad p_3 = (6, 514.85),$
$p_4 = (8, 723.34),$ $\quad p_5 = (10, 968.41),$ $\quad p_6 = (12, 1140),$
$p_7 = (14, 1348.4),$ $\quad p_8 = (16, 1685.5),$ $\quad p_9 = (18, 2083.9)$

are transformed into the final control points:

$q_0 = p_{0,}$ $\quad q_1 = (-0.139, -20.685),$ $\quad q_2 = (-0.177, -26.234),$ $\quad q_3 = p_9$

The comparison of the two graphics is in figure 6.

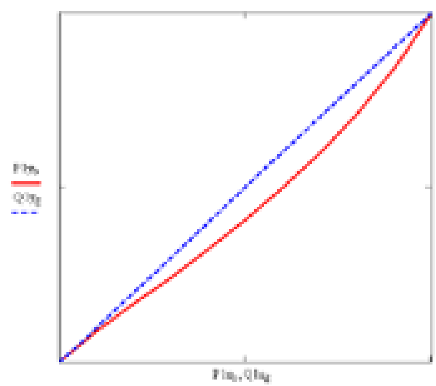

Fig. 6. The case of 9 EMT

Analysing the evolution of the force, depending on the displacement, in table 1, one realizes that the force considerably increases when the number of expanded metal threads increases, if the desired displacement stays constant.

The graphical representations of the dependence between force and displacement, in all the cases under consideration, lead us to some conclusions:

- the force – displacement dependence in case of the deformation of an expanded metal plane plaque made out of expanded metal threads is not linear;
- the force – displacement dependence in case of the deformation of an expanded metal plane plaque made out of expanded metal threads is an increasing function, this monotony property not depending on the dimensions of the plaque;
- the force – displacement dependence in case of the deformation of an expanded metal plane plaque made out of expanded metal threads is better represented by higher degree curves, but the reduction of the degree provides us with mathematical objects easier to use;
- the error of the reduced curves, as we did verify by comparing the intermediate values in table 1 with the computed ones using the

reduced curves, does not exceed 10^{-3}, which is enough acceptable from technical point of view.

As consequence, we elaborated a method of finding enough simple functions, preserving as many as possible properties of the approximated objects, to be convenient for further applications.

4. References

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