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FINITE ELEMENT FOR THE STUDY OF THE FIELD TO THE TEMPERATURE , WITH APPLICATION TO COMPOSITE MATERIALS WITH SHORT FIBERS

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Abstract

In this paper to studing the phenomenon of flow in process of injection in mold, a composites materials reinforced with short fiber using Stephan's problem combined with the homogenization method.
For the numerical procedure we use the finite element method, combined with the bondary element method. The paper concludes with a numerical example analyzing melt filling and fiber orientation.

1.Introduction

The objective of this paper is to calculate of the temperature field in the stage of fill.
A major application of the Stephan's Problem is the analysis of melt filling in injection molded parts with complex geometry. In it's simplest form this method consists of considering a simple-connected or multiple-connected domain bounded in R^3 , a true three-dimensional domain. Since this is an evolution problem, i-e. a time developing phenomenon.
We observe that denoting by Ω the domain we mentioned before, than $\Omega=\Omega(t)$, $t\in$ [t^*_o $_{2,+\infty}$), usually $t^*_o=0$. In the Stephan's Problem $\Omega(t)=\Omega^+(t)\cup\Omega^-(t)$, where the composite material $\Omega^+(t)$ representing the liquid state , respective ,$\Omega^-(t)$,representing the solid (crystallized) state. Fig-1
The two states (phases) $\Omega^+(t)$ and $\Omega^-(t)$ are separated by a surface, mobile in time denoted by $\Gamma^1(t)$. The $\Gamma(t)$ is called the free frontier. We'll see that in its neighborhood there is a band $S_\varepsilon(t)$
of 2ε , $\varepsilon>0$ thickness in what the composite material is half-liquid, respective half-solid.

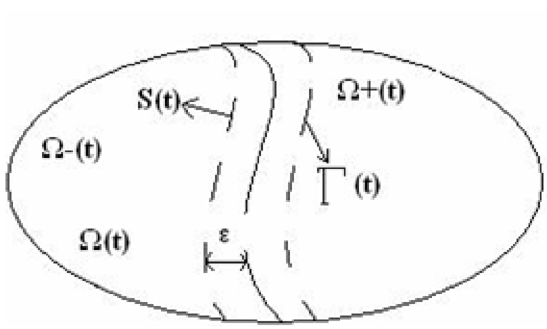

Fig.1.

For simplicity of the report of the Stephan's Problem we'll consider the study of the temperature field from $\Omega(t)$. There fore the field v(t), $t\in[t_o^*, +\infty)$ will be an evolution temperature field which depends on the space $x\in\Omega(t)$, $x = (x_1,x_2,x_3)$ and on the time t.
V: $\Omega(t)$ x[t_o,$+\infty$) $\rightarrow R$, that is $V = v(x, t)= v(x_1,x_2,x_3,t)$.

We suppose there is a constant $V_o=V_o(t_o)$ that is $V_o=V(x_1(t_o),x_2(t_o),x_3(t_o),t_o)$. Considering $t=t_o$ which fixes arbitrarily $\Gamma(t)=\Gamma(t_o)$ and $(x_1(t_o),x_2(t_o),x_3(t_o))\in\Gamma(t_o)$, so that, if $v^+(t)$ respective $v^-(t)$ represents the filed of temperature from the composite $\Omega^+(t)$, respective $\Omega^-(t)$ and:

$$v^-(t_o)\le v_o\le v^+(t_o),t_o\in\left[t_o^+,+\infty\right) \tag{1.1}$$

If we apply to the Fournier- Kirchhoff's equations, associated with the components of the composite Ω(t), the homogenization method [1] or [2] we obtain the equation of the composite Ω(t) of the form :

$$\frac{\partial}{\partial t}(\rho(x,t)v(x,t))=k(x,t)\Delta v(x,t)+f(x,t)+\langle\nabla k(x,t),w(x,t)\rangle,(x,t)\in\Omega(t)x(t_o^*,+\infty), \tag{1.2}$$

Where ρ and κ are the known homogenized coefficients and f represents the external termic loading, also known. In this Stephan's Problem about the usual Henmann – Dirichtet conditions are necessary the Stephan's conditions for the unknown surface $\Gamma(t)$, by:

$$\rho L\cdot V_{\bar{n}}=\left[\ k\cdot\frac{\partial v}{\partial n}\ \right] \tag{1.3}$$

The right side of the relation (1.3) represents the "jump" of the flow . If $h:\Omega(t)x\left[t_o^*,+\infty\right)\to R$ and $\Gamma(t)\subset\Omega(t)$ is the free surface arbitrarily fixed for the moment $t\in\left[t_o^*,+\infty\right)$,then the jump of h relative to $\Gamma(t)$ is defined by $[h]=\lim\limits_{\substack{(x,t)\to\Gamma(t)\\ v<v*}}-\lim\limits_{\substack{(x,t)\to\Gamma(t)\\ v>v*}}$

where v_* is the constant temperature, and V is the field of temperature from Ω(t).

2.The variational equations of the temperature and pressure field

In the assumptions exposed by (1.1), (1.2) and (1.3) about the temperature field v(x, t) we suppose besides that the function v(x, t) is an element of the Sobolev space $H_2(\Omega(t))$, $t\in[0, T]$, [3], then the Fourier-Kirchhoff's equation has an equivalent form in Sobolev sense, given by the following variational equation:

$$\begin{aligned}&\iiint_{\Omega(t)}dx_1dx_2dx_3\int_0^T\left\{k\cdot\nabla v(x,t)\nabla\varphi(x,t)-U(x,t)\cdot\frac{\partial f(x,t)}{\partial t}-f(x,t)\cdot\varphi(x,t)\right\}dt=\\&\iiint_{\Omega(0)}U(x,0)\cdot f(x,0)dx_1dx_2dx_3,\forall\varphi\in H_2^0(\Omega(t))\end{aligned} \tag{2.1}$$

In this variational equation there are the following elements: the function $\cup(x_1t)$, which is in fact a multifunction, generally defined by:

$$U=\begin{cases}\int_0^{v(x,t)}c_+(r)dr,\text{if}v<v_*\\ [-1,0],\text{if}v=v_*\\ \int_0^{v(x,t)}c\_(r)dr,\text{if}v>v_*\end{cases} \tag{2.2}$$

where c_+ is the thermic capacity at the constant volume corresponding to the discontinuous composite $\Omega^+(t)$, and c is the thermic capacity at the constant volume of the gas $\Omega^-(t)$. These constants depend on the norm of the position vector of the point x = (x_1, x_2, x_3) from $\Omega(t)$. The integral refers to the variable temperature t = r for each x arbitrarily fixed. The constant L > 0 represents the latent temperature and the interval [-L, 0] is the zone of the half solid discontinuous composite. This interval [-L, 0] permits a study of the values for $\cup\in$[-L, 0] when V = V_*.
As we'll see in the following section in (2.1) the unknown is the field v (x_1t) while the function φ is free and will be particularized in order to apply the finite element method.
The constant φ representing the density of Ω(t) is included in the boundary conditions.
The knowledge of the temperature field is very important, because by means of its we can determine the thermic flux. Q, given by:

$$q = -K\nabla V \tag{2.3}$$

In this relation appears explicitly the thermic conductivity K.
About the pressure and velocity field of the discontinuous composite there are information in [4] and [5].
The idea of this paper is to stand out the formulation of the problem injection, in view of Stephan's problem.

3.The finite element representation

For a finite element discretization of the domain $\Omega^+(t)$ in geometrical finite elements $\{\Omega^e(t)\}^n{}_{e=1}$, for each geometrical finite element $\Omega^e(t)$ we'll write a base of shape functions, which in a first approximation will depend only on space, i.e. an autonomous base $\{\psi_k^{(e)}(x_1,x_2,x_3)\}_{k=1}^{Ne}$ where Ne represents the local knots number of the element $\Omega_e(t)$.

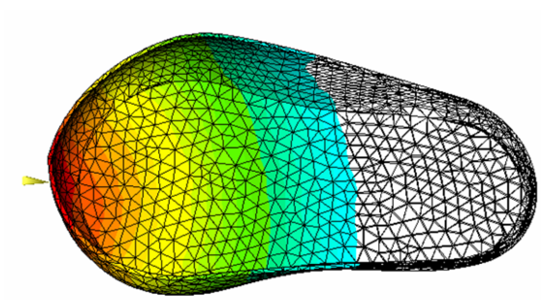

Fig.2

Consequently, a local representation of the temperature field

$$\tilde{v}_e(x,t) = \sum_{k=1}^{Ne} v_k(t)\psi_k^e(x_1,x_2,x_3), \tag{3.1}$$

where v_k (t) represents the temperature in the local knot K from the element Ω_e (t), which is a function of the time t.
Following the technique of the finite element discretization the variational equation (2.1) is transformed in a differential equation system, with the unknowns $V_k(t)$, k=1÷n. Here n represents the number of global knots of the finite element .
Usually, this differential equations system is numerically integrated.

The boundary element method is used at every step of the time. In numerical process of the approximate solution of the differential equations system in V_k (t), we discretise the [O,T] interval in a division Δ_m, $\Delta_{m:}$ $_0$=$t_0<t_1$-------$<t_i<t_{iN}$-----$<t_m=T$ and determine the approximations $W_{k,i}$ on each time interval[t_i,t_{iN}].
At the moment t_i we have to determine the free frontier .
This surface we approximate starting with the Stephan's condition (2.3).
The approximation of the $_{M(}t_{i)}$we obtain by a "boundary element" procedure, it can be consulted in [4].
The proposed application follows this mathematical theory.

4.Simulation of the discontinuous composite fluid flow

The analyses of the melt plastic material in injection molded cam be of the type:

- Full flow
- Filling only
- Runner balance
- Molding window
- Gate location

In this paper we used only the first and the second option analysis. In order to analyze the flow of plastic material during it's injection in the mold it's necessary to cover several stages;(fig 3

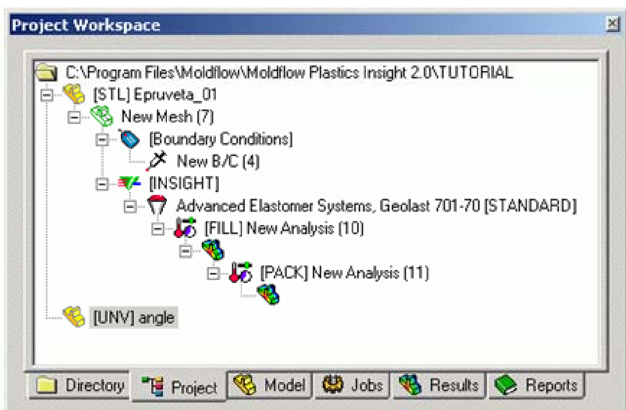

Fig.3 Program files / moldflow plastics

1. The import of 3D models of the piece(test tube)

The geometrical model of the piece (fig.3) to be analyzed can be realized directly in Mold flow, using the module MPI/Modeler or in the CAD 3D program , afterwards imported in MDI/STUDIO module as a file STL or IGES.

2. The discretization of geometrical model (New Mesh)

It's a very important stage of the simulating process, because it can appear various deficiencies of the mash, for example: breaches , superposition of the elements, unorientated elements. This deficiencies doesn't permit the realization of the simulating and appear messages of errors.

3. The selection of the boundary conditions

In this stage we choose the point of the injection. The place will be represented by a yellow knot on model surface.

4. Stualysis Preparation Wizard

In this stage take place the choice of the material for the piece from the data base of the soft, the choice of the analysis type (Flow , Gptinum profile analysis, Cool, Warp or Stress) Choosing the Flow option, further will choose the oprion Filling only and the necessary parameters for the injection.

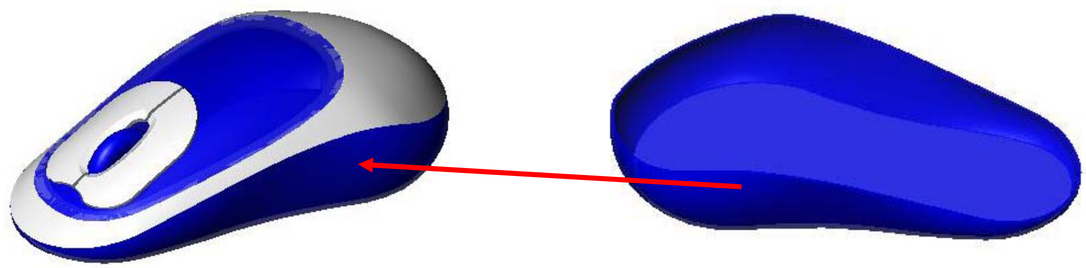

Fig. 4 Test tube from plastic material, the model to be studied

In this simulating we'll obtain the the flow front temperature.
The diagram of the flow fronts temperature(fig.5 presents the temperature distribution when the flow front reaches a specified point. The graple can be obtaind at the end of the analysis ar at a specified time during the analysis. Ts recomanded a small variation of the flow fronts temperature from the first point of the cavity until the last filled point.

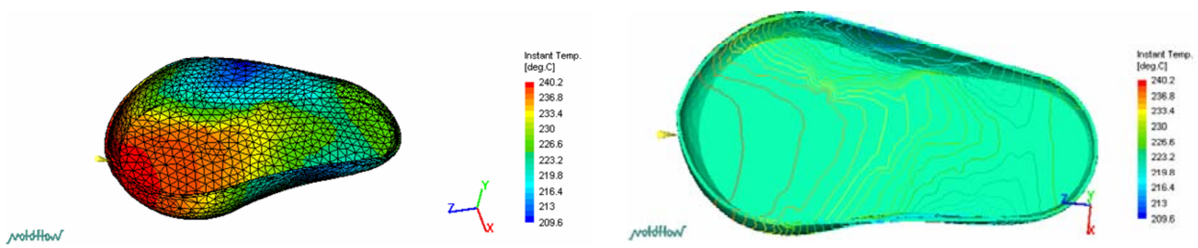

Fig. 5The flow fronts temperature diagram

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