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HYDRAULIC CALCULUS OF A LOOP WITH A LARGE NUMBER OF BARS

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Abstract: In the work it is presented a hydraulic calculation of the loop with „*N*" pipes. In a concret way it's considered the number *N=10*. We take the flows through the loop's section in a quadratic turbulent regime where the head loss is proportionate with quadratic flows. The autors contribution consist in the original calculation algorithm.

1. Theoretical Basis

We consider the situation in which there are known the caracteristics of the pipes which forms the loop. The water 's volume entering into loop, is equal with the water 's volume going out from nodes.

The loop has *N* pipes, limitated by *N* nodes, so there is only one node where water enters into the network and *N-1* nodes through which the water goes out.

The nodes are numbered from 1 to *N*. Numer 1 is the node through the water enters, from the network and the final node (*N+1)* coincides with the first one (*N=1*).

The numbering is made in a conventional way, being positive. This method is more avantageous than the usual iterative one as long as *N* is bigger.

It is obvious that the continuity flows equation should be satisfacted:

$$Q_1 = \sum_{i=2}^{N} Q_i \qquad (1)$$

The volume that flows through the pipe which connect node *i* with node *i+1* is noted $Q_{i,i+1}$ and represented an unknown of the problem, in all being *N* unknowns.

We have a number of *N* equations, from which N-1 are continuity equations and one a dynamic equation.We have also *N* unknows, representing the flows through pipes. According to it, the sum of head loss should be zero in the whole loop.

It's used the following way to resolve the problem: we considere as an unknown (*x*) the flow Q_{12}, flowing between the nodes number one and number two. With the continuity equations are expressed all the other volumes in accordance with *x*.

So, the flows are noted as:

$$Q_{23} = x - Q_2$$
$$Q_{34} = x - Q_2 - Q_3$$
.... (2)
$$Q_{i,i+1} = x - \sum_{j=2}^{i} Q_j$$

We calculate the head loss with usuall formula, using hydraulic resistance modulus $M_{i,i+1}$:

$$h_{r_{i,i+1}} = M_{i,i+1} Q_{i,i+1}^2 = M_{i,i+1}\left(x - \sum_{j=2}^{i} Q_{j,j+1}\right)^2 \qquad (3)$$

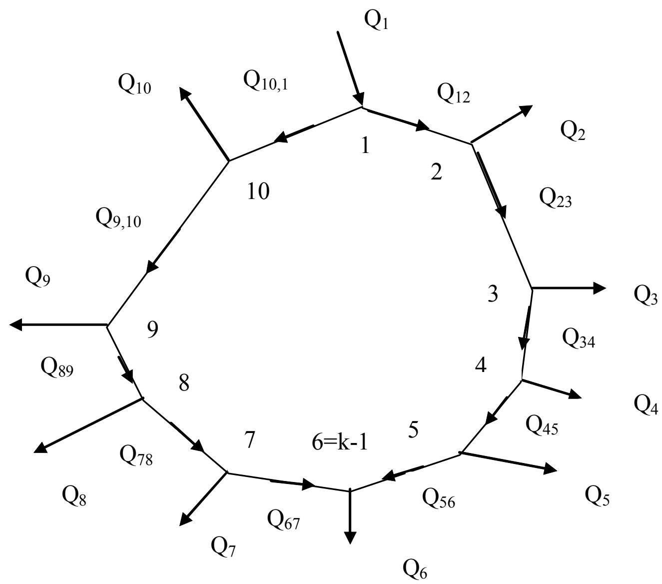

Fig.1.Loop with N bars

The first problem in direct resolving (without using iterative method) is to establish the node *k* where is modified the direction of the water in the loop. In the figure, for example *k-1=6*. For this, it's considered the head loss function $f_r(x)$:

$$f_r(x)=\sum_{i=1}^{N}\left(\pm h_{r_{i,i+1}}\right)=\sum_{i=1}^{k-1}h_{r_{i,i+1}}-\sum_{i=k}^{N}h_{r_{i,i+1}} \tag{4}$$

It is obvious that the solution of the problem will be for $f_r(x)=0,$ if the *k* parameter's value is correct determinated. So, we write:

$$f_r(x)=\sum_{i=1}^{k-1}\left[M_{i,i+1}\left(x-\sum_{j=1}^{i}Q_j\right)^2\right]-\sum_{i=k}^{N}\left[M_{i,i+1}\left(x-\sum_{j=1}^{i}Q_j\right)^2\right] \tag{5}$$

For simplification we note:

$$a_i=\sum_{j=2}^{i}Q_j \;\; ; \; a_1=0 \tag{6}$$

So, the head loss function takes this form:

$$f_r(x)=\sum_{i=1}^{k-1}\left[M_{i,i+1}(x-a_i)^2\right]-\sum_{i=k}^{N}\left[M_{i,i+1}(x-a_i)^2\right] \tag{7}$$

Developing previous expression, the function can get the canonical form of the quadratic function:

$$f_r(x)=Ax^2-2Bx+C$$
$$A=\sum_{i=1}^{k-1}M_{i,i+1}-\sum_{i=k}^{N}M_{i,i+1}$$
$$B=\sum_{i=2}^{k-1}a_iM_{i,i+1}-\sum_{i=k}^{N}a_iM_{i,i+1} \tag{8}$$

$$C = \sum_{i=2}^{k-1} a_i^2 M_{i,i+1} - \sum_{i=k}^{N} a_i^2 M_{i,i+1}$$

Now, some mathematical appreciations of head loss function are made. These will permite to determinate the value of the parameter *k* .

If variable *x* has a value bigger than the exact one, the volumes and head loss will increase, so the function will become reciprocal positive. This means:

$$x > Q_{12} \Rightarrow f_r(x) > 0 \qquad (9)$$

$$x < Q_{12} \Rightarrow f_r(x) < 0 \qquad (10)$$

In the other hand, considering the a_i constant's value increase with the increase of *i* parameter, we can say that *B* and *C* constants will be negative while the *A* constant could be positive or negative, being neccesary a suplimentary analyse.

The funcion $f_r(x)$ generally looks like the graphic of the figure 2. If the subscript „*i*" becomes bigger, from equation (*7*) it resuts that $f_r(x)$ also becomes bigger and for $x = Q_{12}$, $f_r(x) = 0$. The maximum value of the subscript for which *A<0* shows the node where changes the flow in the loop (the node *K-1*). In generally , are obtained two positive solutions but only the solution with minimal value is correct. The singular case *A=0* is represented in the scheme. In this situation head loss function degenerates in a straight line.

Calculations can be introduce in a table. If the *k-1* parameters value is determinated (the node where the water's direction is modifeied in the loop) a_i, *A, B, C* constants values are counted according to the previous relations.

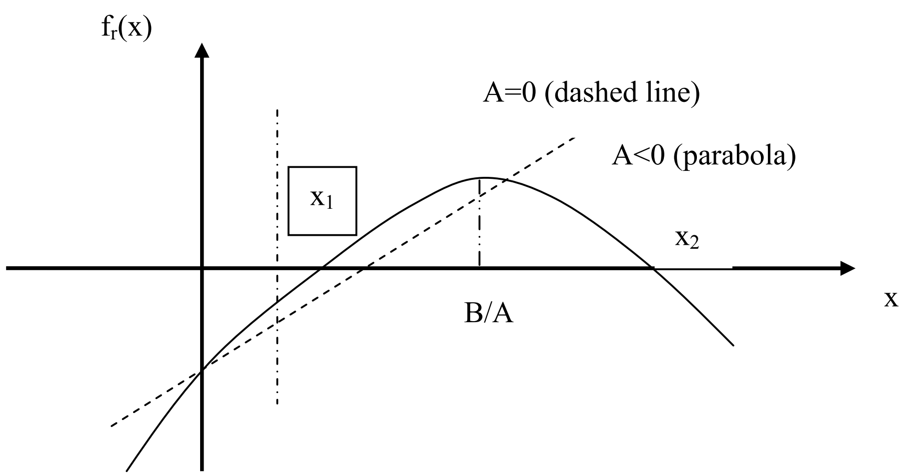

Fig.2.Function $f_r(x)$

Finally

$$Q_{12} = \frac{B \pm \sqrt{B^2 - AC}}{A} \qquad (11)$$

If A=0 , the solution is:

$$Q_{12} = \frac{C}{2B} \qquad (12)$$

2. Numerical Example

It's a loop having 10 bars, with following characteristics:

Tabel 1. Initial date

i	1	2	3	4	5	6	7	8	9	10	$\sum_{2}^{10} Q_i$
$\lvert Q_i \rvert$ (l/s)	0	26	12,5	18,3	13,8	13,5	8	11,2	22,2	1,2	136,7
$l_{i,i+1}$ (m)	950	450	800	230	750	400	300	820	500	620	5820
$d_{i,i+1}$ (mm)	1000	800	700	500	350	300	300	500	800	900	-
$M_{i,i+1}$ (s^2/m^5)	1,415	2,209	8	13,846	303	367,2	275,4	49,364	2,455	1,624	1024,51
a_i (l/s)	0	26	38,5	56,5	70,	84,1	92,1	103,3	125,5	136,7	-

Table 2. Determination of the node *k-1*

i	2	3	4	5	6	7
$\sum_{i=1}^{k-1} M_{i,i+1}$	1,415	3,624	11,624	25,470	328,470	695,670
$\sum_{i=k}^{N} M_{i,i+1}$	1023,098	1020,889	1012,889	999,043	696,043	328,843
f_r	-1021,683	-1017,265	-1001,265	-973,573	-367,573	+366,827

k-1=6; A=-367,573

Tabel 3. Determination of the constant B and C

Node	2	3	4	5	6	$\sum$	7	8	9	10	$\sum$
a_{i+1}	26	38,3	56,5	70	84,1	-	92,1	103,3	125,5	136,7	-
$a_{i+1}M_{i,i+1}$	0,057	0,308	0,786	21,391	22,543	30,881	25,364	5,099	0,308	0,222	61,875
$a_{i+1}^2 M_{i,i+1}$	0,001	0,011	0,044	1,510	1,568	2,597	2,336	0,526	0,038	0,030	5,528

B=30,881-61,875= -30,994 ; C=2,597-5,528= -2,931

Tabel 4. Finaly results

i,i+1	1-2	2-3	3-4	4-5	5-6	$\sum$
$Q_{i,i+1}$ (l/s)	81,03	55,05	42,55	24,25	10,45	-
$h_{r_{i,i+1}}$ (m)	0,009294	0,006696	0,01448	0,008142	0,03308	0,0717
i,i+1	6-7	7-8	8-9	9-10	10-1	$\sum$
$Q_{i,i+1}$ (l/s)	3,05	11,05	22,25	44,45	55,65	-
$h_{r_{i,i+1}}$ (m)	0,003416	0,03362	0,02443	0,004851	0,005031	0,0713

3. Final Considerations

In this paper, we peent a simple algoritm for hydraulic calculu of a loop. The results can be presented in a table form and it's realised following these fie steps:

a) It's made up a table form with known characteristics of the loop (tabel 1);
b) The node where the flow changes the direction and the constant A is determinated (tabel 2);
c) Constants B,C value are determinated (tabel 3);
d) The final resut concerning the flows through the pipes are determinated (tabel 4);
e) Head losses on every bar are determinated and verified the condition $\sum h_{r_{i,i+1}} = 0$ on the loop.

The autors have elaborated an automatic calculation programm with a graphic interface that can be used easily, which will be presented in another paper.

4. Bibliography

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